Temporal Mechanics

Edwin Alan Pease
(Dated: April 25, 2026)

By identifying Schrödinger phase, proper time, and internal holonomy as related aspects of a single temporal structure, Temporal Mechanics proposes a common geometric origin for the distinct notions of time appearing in quantum theory and relativity. In this framework, clocks unwind a hidden periodic structure whose abelian holonomy is shared by quantum phase and proper time. The internal sector is modeled as a compact three-torus $\mathcal{M}_\tau \cong \mathbb{T}^3$ equipped with a $U(3)$ bundle whose determinant line is identified, by a chosen phase lock, with the auxiliary $U(1)$ line of a $\mathrm{Spin}^c(1,3)$ structure on an emergent Lorentzian spacetime $\mathcal{M}_4$. The traceless sector carries nonabelian geometric structures allowing Higgs-like and flavor degrees of freedom which are interpreted as modes of the internal connection. The induced internal Dirac spectrum provides geometric mass scales. This reinterpretation of proper time, gauge structure, and mass as related manifestations of a common temporal geometry removes the tension between quantum mechanics and relativity.

I. INTRODUCTION

All abelian structures in Temporal Mechanics (TM) are taken to arise from a single temporal holonomy line L_τ. They are not introduced as independent $U(1)$ sectors, but functorial realizations of the same underlying temporal phase in the Schrödinger phase of quantum evolution, the determinant line of the internal $U(3)$ bundle, the auxiliary $U(1)$ line in the external Spin^c structure, and the line-bundle twists that furnish effective hypercharges of matter fields.

$\mathcal{M}_4$ denotes the emergent Lorentzian spacetime with metric $g_{\mu\nu}$ and signature $+---$, and $\mathcal{M}_\tau$ denotes the compact internal temporal manifold specialized to $\mathbb{T}^3$. Let $\mathcal{M}_7 := \mathcal{M}_4 \times \mathcal{M}_\tau$ be the global product with projections $\pi_4 : \mathcal{M}_7 \to \mathcal{M}_4$ and $\pi_\tau : \mathcal{M}_7 \to \mathcal{M}_\tau$. Greek indices $\mu, \nu = 0, 1, 2, 3$ refer to $\mathcal{M}_4$ with coordinate x^μ on $\mathcal{M}_4$, Latin indices $a, b = 1, 2, 3$ to $\mathcal{M}_\tau$, and capital indices $A, B = 0, \ldots, 6$ to $\mathcal{M}_7$. The internal rank-3 Hermitian bundle is $E \to \mathcal{M}_\tau$. Its determinant line is $L := \det E$, and $L_{\mathrm{Spin}^c} \to \mathcal{M}_4$ is the auxiliary line of the external $\mathrm{Spin}^c(1,3)$ structure. Let s be the worldline parameter, $t = x^0$ be coordinate time, τ be proper time, and $\tilde{\theta}$ be the lifted phase of the locked temporal $U(1)$ line. Finally, let $\hbar = c = 1$.

"The theory of relativity is concerned with the connection between the descriptions of phenomena as viewed from two different coordinate systems which are in motion relatively to each other" [1]. In TM the compact manifold is temporal and its determinant $U(1)$ is locked to proper time, so that spacetime geometry, gauge charges, and rest masses are all geometric manifestations of internal time. Relativity is then a statement of connections to the temporal manifold. Einstein's original postulates of special and general relativity [1–4] can be reinterpreted with temporal geometry.

Postulate 1: Time oscillation is universal, where all physical system possess intrinsic motion within the three-dimensional temporal manifold $\mathcal{M}_\tau$ with coordinates θ^a, defining the system's rest mass and inertial identity.

Postulate 2: All physical trajectories are dictated by curvature within the total pseudo-Riemannian manifold $\mathcal{M}_7$, with temporal structure encoded in the internal geometry of $\mathcal{M}_\tau$ and associated bundle.

Postulate 3: The generalized temporal bundle

$$\mathcal{E}_\tau = T\mathcal{M}_\tau \oplus T^*\mathcal{M}_\tau, \tag{1}$$

and the observed causal structure of spacetime is described by the $U(1)$ phase locked fiber

$$\mathcal{G} = U(3) \times_{U(1)} \mathrm{Spin}^c(1,3). \tag{2}$$

Postulate 4: Temporal coherence and gauge information propagate at a universal limiting speed. When projected onto $\mathcal{M}_4$, this limiting speed is the speed of light, preserving standard relativistic causality.

Postulate 5: The curvature of the external spacetime manifold $\mathcal{M}_4$ is sourced by the energy–momentum associated with the internal geometry of the temporal manifold $\mathcal{M}_\tau$ through the phase-locked determinant line and the induced external Lorentzian geometry.

Section II defines time from first principles, showing how periodicity and phase evolution define temporal dynamics. Section III provides the global $U(1)$ symmetry which encodes time evolution. Section IV generalizes this to $U(3)$ to account for flavor and charge. Section V introduces the extended symmetry group $SO(3,3)$. Section VI demonstrates how both spacetime and the SM gauge symmetries arise through phase locking. Section VII formalizes the product manifold by defining its bundles and forms. Section VIII provides the particles of the SM. Section IX addresses how mass and energy emerge from the spectral structure of the Dirac operator. Section X provides rest mass eigenvalues and Section XI discusses mixing and Higgs dynamics using holonomy and curvature.

II. TIME

Definition: A clock is any physical system whose state evolves with stable, periodic trajectories on the temporal manifold $\mathcal{M}_\tau$, such that each complete cycle

 DOI: 10.5281/zenodo.19897815

defines a unit of experienced time in the emergent external spacetime $\mathcal{M}_4$.

Clocks are the physical instruments by which time is measured. Such has been the advancement in the fractional error of clocks that one may now measure time to an error of about one second since the universe began. Clocks provide the description for time, and every clock, no matter how it is constructed, experiences Einstein's special and general relativity [1, 5, 6].

Early mechanical clocks emerged in the late 13th century in Europe driven by weights and later in the 15th century by springs. The cornerstone of introductory physics is the pendulum clock invented in 1656 by Christiaan Huygens. This simple clock helps provide the description of motion necessary to characterize time.

The mechanical pendulum clock works by counting the revolutions of oscillation. Looking at the pendulum in configuration space, the Lagrangian can be written as

$$\mathcal{L} = \mathcal{K} - \mathcal{V}, \tag{3}$$

where $\mathcal{K}$ is the kinetic energy and $\mathcal{V}$ is the potential energy. A simple pendulum consists of a mass m attached to a rod or string of length ℓ swinging under gravity g, where its position is described using a single generalized coordinate of the angle from vertical φ. The Lagrangian is $\mathcal{L}(\varphi, \dot{\varphi}) = \frac{1}{2}m\ell^2\dot{\varphi}^2 - mg\ell(1-\cos\varphi)$. Using $\cos\varphi \approx 1 - \varphi^2/2$ yields the harmonic expression

$$\mathcal{L} \approx \tfrac{1}{2}m\ell^2\dot{\varphi}^2 - \tfrac{1}{2}mg\ell\,\varphi^2 = \frac{\beta}{2}\,\dot{\varphi}^{\,2} - \frac{\beta\omega^2}{2}\,\varphi^2, \tag{4}$$

where $\beta = m\ell^2$ is interpreted as the phase inertia and ω is the angular frequency. From the principle of least action, the equation of motion from Euler-Lagrange of the harmonic form is

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \frac{\partial \mathcal{L}}{\partial \varphi} = 0, \tag{5}$$

yielding

$$\ddot{\varphi} + \omega^2\varphi = 0. \tag{6}$$

Then the solution to the equation of motion for the harmonic oscillator is

$$\varphi(t) = \mathcal{A}\cos(\omega t + \theta_0), \tag{7}$$

where $\omega = \sqrt{\dfrac{g}{\ell}}$.

t is the external parameter labeling an observer's worldline, and it is the universal cover coordinate of the internal clock phase. For the ideal clock with fundamental period $T = 2\pi/\omega$, the physical state after time t can be characterized by its phase $\theta(t) = \omega t$ where mod $2\pi \in S^1$. The map $t \in \mathbb{R} \mapsto e^{-i\theta(t)} = e^{-i\omega t} \in U(1)$ is a group homomorphism from the additive group $(\mathbb{R}, +)$ of time translations to the multiplicative group $(U(1), \cdot)$, where the composition of time intervals corresponds to multiplication of phases, $e^{i\omega t_1}e^{i\omega t_2} = e^{i\omega(t_1+t_2)}$. For a periodic system, physical states are identified under $t \sim t + nT$, so the evolution factors through the quotient $\mathbb{R}/T\mathbb{Z} = S^1$, which is the group manifold of $U(1)$.

In the Lagrangian formulation, second-order differential equations in time occur in the Euler-Lagrange equation which reflects how dynamics evolve through extremal action. The coupled first–order equations governing the dynamics arise from the Hamiltonian,

$$\mathcal{H} = \mathcal{K} + \mathcal{V}. \tag{8}$$

The corresponding first-order equations are Hamilton's equations are

$$\dot{q} = \frac{\partial H}{\partial p_q}, \qquad \dot{p}_q = -\frac{\partial H}{\partial q}. \tag{9}$$

For the pendulum, $\mathcal{H} = \dfrac{p_\varphi^2}{2m\ell^2} + mg\ell(1-\cos\varphi)$, the harmonic form becomes

$$\mathcal{H}(\varphi, p_\varphi) = \frac{p_\varphi^2}{2\beta} + \frac{\beta\omega^2}{2}\,\varphi^2. \tag{10}$$

The parameter β measures how resistant the internal phase φ is to changes in its rate of winding, where the conjugate momentum

$$p_\varphi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \beta\,\dot{\varphi} \tag{11}$$

is the phase momentum, and in the free-rotor limit,

$$\mathcal{L}_{free} = \frac{\beta}{2}\dot{\varphi}^2. \tag{12}$$

This momentum is conserved, and its quantization corresponds to discrete winding numbers of the internal phase on S^1. In TM this simple clock is promoted to a field theory on the temporal manifold $\mathcal{M}_\tau$, where the phase coordinates φ^a become local angular coordinates on a three-torus $\mathbb{T}^3$ and the phase inertia generalizes to a tensor β_{ab} proportional to the internal metric g_{ab}, where the conserved phase momenta

$$p_a = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}^a} = \beta_{ab}\dot{\varphi}^b \tag{13}$$

encodes how a field winds in the multiple temporal directions. After quantization these p_a become the discrete Kaluza-Klein momenta associated with angles θ^a and charges associated with the compact temporal dimensions identified by invoking Noether's theorem with symmetry generated currents in temporal space [7].

For classical systems like the pendulum, the phase of oscillation can be interpreted as a coordinate on a compact direction, such that each oscillation corresponds to a completed loop through $\mathcal{M}_\tau$. For the atomic clock, the time-dependent Schrödinger equation is given by

$$i\frac{\partial}{\partial t}\Psi(\mathbf{r}, t) = \hat{H}\Psi(\mathbf{r}, t). \tag{14}$$

This can be solved by the separation of variables, where

$$\Psi(\mathbf{r},t)=\psi(\mathbf{r})\chi(t),\quad \chi(t)=Ae^{-iE_nt}. \tag{15}$$

The Hamiltonian is the total energy with states defined as

$$\hat{H}\psi_n(\mathbf{r})=E_n\psi_n(\mathbf{r}). \tag{16}$$

For oscillatory systems including pendula, atomic transitions, and spinor precession, the time dependent Schrödinger's equation reduces to phase evolution with a defined frequency and is composed of an amplitude and a phase [8]. The Hamiltonian evolution in external time is a shadow of the winding in internal time and serves as a bridge between temporal geometry and external time measurement.

For an energy eigenstate with energy E_n, the time evolution is

$$\chi(t)=e^{-iE_nt}\chi(0), \tag{17}$$

where the physical state at time t differs only by a global phase in $U(1)$. As a temporal evolution operator, time advances regardless of the direction of the winding. The subgroup of the full unitary evolution corresponding to this phase is

$$\hat{U}(t)=e^{-iE_nt}\in U(1)\subset U(\mathbb{H}), \tag{18}$$

and the phase is defined modulo 2π or on S^1, which is a central subgroup of the unitary group acting on the Hilbert space.

Following from variation of the action with respect to the phase field φ on $(\mathcal{M}_4,g_{\mu\nu})$, the scalar-field Lagrangian is

$$\mathcal{L}=\frac{\beta}{2}\,g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi-\frac{\beta\omega^2}{2}\,\varphi^2, \tag{19}$$

with action

$$S_\varphi=\int_{\mathcal{M}_4}d^4x\,\sqrt{-g_4}\,\mathcal{L}. \tag{20}$$

Stationarity $\delta S_\varphi/\delta\varphi=0$ gives the curved-space Euler–Lagrange equation

$$\frac{1}{\sqrt{-g_4}}\partial_\mu\left(\sqrt{-g_4}\,\beta\,g^{\mu\nu}\partial_\nu\varphi\right)+\beta\omega^2\varphi=0, \tag{21}$$

which reduces to $\ddot{\varphi}+\omega^2\varphi=0$ in flat space and in the homogeneous limit.

Each oscillation of a physical system, like an atomic transition, maps onto periodic trajectories in the temporal manifold, establishing a direct correspondence between internal geometry and experimental time intervals. These periodicities trace closed loops in $\mathcal{M}_\tau$, linking observable time intervals in $\mathcal{M}_4$ to geometric properties of the temporal manifold. This defines the external time parameter $t\in\mathbb{R}$, which labels points on an observer's worldline, from the internal phase $\theta\in S^1$ of a clock. The phase lives on a circle, and its evolution is described by a $U(1)$ action, while t is the lift of that circular motion to the universal cover $\mathbb{R}$.

Across classical mechanics, quantum theory, and field theory, clocks manifest as oscillatory systems governed by dynamics rooted in time. This time is a manifold with a complex phase evolution in quantum systems. The system's state is embedded in this temporal manifold making time more than an external parameter.

III. $U(1)$

The $U(1)$ group is the minimal symmetry that encodes the phase evolution of a quantum system[8]. TM assumes a single underlying temporal holonomy line $L_\tau\to\mathcal{M}_7$ with structure group $U(1)$. The Schrödinger phase of quantum evolution, the determinant line of the internal $U(3)$ bundle, the auxiliary $U(1)$ line of the external $\mathrm{Spin}^c(1,3)$ structure, and the line-bundle twists carried by matter fields are taken to be associated realizations of this temporal $U(1)$, not independent abelian sectors. The phase lock is the geometric identification that ties these realizations together.

A clock modeled by free motion on S^1 with angular coordinate $\theta\sim\theta+2\pi$ and free-rotor Lagrangian has a shift symmetry $\theta\to\theta+\epsilon$ yielding conserved momentum $p_\theta=\beta\dot{\theta}$. It is useful to distinguish the signed lifted phase from the operationally accumulated clock time

$$\tilde{\theta}(s)=\frac{1}{2\pi}\int_0^s\dot{\theta}(u)\,du,\quad t(s)=\frac{1}{2\pi}\int_0^s|\dot{\theta}(u)|\,du, \tag{22}$$

where the clock time reduces to $\dfrac{|\dot{\theta}|}{2\pi}s$ when p_θ is constant. Reverse winding changes the sign of $\tilde{\theta}$ but not of the accumulated time t.

$U(1)$ is the group of complex numbers under multiplication,

$$U(1)=\left\{e^{-i\theta}\mid\theta\in[0,2\pi)\right\}. \tag{23}$$

For any continuous path, $\mathcal{C}:[0,T]\to S^1$, the lifted path is $\tilde{\mathcal{C}}:[0,T]\to\mathbb{R}$, and $U(1)$ is a compact and periodic group, where the winding number is defined as

$$w[\mathcal{C}]=\frac{1}{2\pi}\int_0^T\frac{d\theta}{ds}(s)\,ds. \tag{24}$$

The observable time t is a positively oriented monotonic accumulation over internal time.

The real line $t\in\mathbb{R}$ parameterizes the covering space of these windings and the Hamiltonian determines how the wavefunction winds around the circle fiber as time progresses and is the generator of the Lie group $U(1)$. The total Hamiltonian $\hat{H}$ is self-adjoint and $-i\hat{H}$ lies in the Lie algebra $\mathfrak{u}(n)$, generating a one-parameter subgroup

$\hat{U}(t) = e^{-i\hat{H}t} \in U(n)$ acting on a finite-dimensional internal Hilbert space. Its central $U(1)$ factor corresponds to the overall phase generated by the trace part of $\hat{H}$, while the traceless part lives in $\mathfrak{su}(n)$ with

$$-i\hat{H} \in \mathfrak{u}(\mathcal{H}), \text{ with } \det \hat{U}(t) = e^{-i\,\mathrm{tr}\hat{H}\,t} \in U(1). \quad (25)$$

The global phase of $\hat{U}(t)$ given by $\frac{\mathrm{tr}\,\hat{H}}{n}$ defines a central $U(1) \subset U(n)$ that encodes the total energy of the system. When $\hat{H}$ acts as $E_\tau \mathbf{1}$ on the subspace $\mathcal{H}_\tau$, the dynamics reduce to a pure $U(1)$ rotation generated by $-i\hat{H}_\tau \in \mathfrak{u}(1)$.

Let $P \xrightarrow{\pi} \mathcal{M}_\tau$ be a principal $U(1)$ bundle with global connection one-form $\mathcal{A} \in \Omega^1(P, \mathfrak{u}(1))$. In a local trivialization $s_\alpha : U_\alpha \to P$, the local gauge potential is

$$A_\alpha = s_\alpha^* \mathcal{A} \in \Omega^1(U_\alpha, \mathbb{R}), \quad (26)$$

and on overlaps $U_\alpha \cap U_\beta$ the local potentials differ by a gauge transformation. The curvature is the globally defined closed two-form $F \in \Omega^2(\mathcal{M}_\tau, \mathbb{R})$ characterized locally by

$$F|_{U_\alpha} = dA_\alpha. \quad (27)$$

The topology of the $U(1)$ bundle is encoded by the first Chern class

$$c_1(P) = \left[\frac{1}{2\pi} F\right] \in H^2(\mathcal{M}_\tau, \mathbb{Z}), \quad (28)$$

whose pairing with a closed 2-cycle $\Sigma \in H_2(\mathcal{M}_\tau, \mathbb{Z})$ gives the quantized flux condition

$$\frac{1}{2\pi} \int_\Sigma F \in \mathbb{Z}. \quad (29)$$

For a loop $\mathcal{C}$ contained in a local chart with $\mathcal{C} = \partial\Sigma$, Stokes' theorem gives

$$\oint_\mathcal{C} A_\alpha = \int_\Sigma F,$$

and the associated Wilson loop is

$$W(\mathcal{C}) = \exp\Big(-i \oint_\mathcal{C} A_\alpha\Big) = \exp\Big(-i \int_\Sigma F\Big). \quad (30)$$

For a flat connection, holonomy depends only on the homotopy class of the loop. In general, it depends on the connection along the loop, and Stokes' theorem relates it locally to the curvature on a spanning surface when one exists, while the integral cohomology class $[F/2\pi]$ records the global topological sector [9–13]. Equivalently, a spinor transported around $\mathcal{C}$ picks up the geometric phase $\psi \mapsto W(\mathcal{C})\psi$, and the resulting charge quantization is the bundle content of the Dirac–Wu–Yang condition on a nontrivial 2-cycle [9, 14–17].

For the $U(1)$ group, the global phase symmetry is $\psi \to e^{iq_1\alpha}\psi$. The covariant derivative is

$$D_\mu = \partial_\mu + iq_1 A_\mu \quad (31)$$

with the gauge field

$$A_\mu \to A_\mu - \partial_\mu \alpha \quad (32)$$

and has the local symmetry $\psi \to e^{iq_1\alpha}\psi$. The conserved current is

$$J^\mu = q_1 \bar{\psi}\gamma^\mu\psi \quad (33)$$

and the field strength is

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (34)$$

The commutator of the covariant derivative is

$$[D_\mu, D_\nu] = iq_1 \mathcal{F}_{\mu\nu}, \quad (35)$$

where the generators T^A are Hermitian, the gauge potential A_μ has real components A_μ^A, and the field $\mathcal{F}_{\mu\nu}$ is Hermitian.

The unitary group $U(1)$ appears in multiple foundational contexts of modern physics as the gauge group of electromagnetism and hypercharge [18–20]. $U(1)$ emerges as the residual symmetry of temporal geometry, and acts as the generator of phase evolution. This geometerizes both quantum time evolution and gauge interactions under a single temporal symmetry.

The global $U(1)$ symmetry defines a principal fiber bundle over the temporal manifold $\mathcal{M}_\tau$, with group $U(1)$, total space P, and projection map $\pi : P \to \mathcal{M}_\tau$,

$$U(1) \hookrightarrow P \xrightarrow{\pi} \mathcal{M}_\tau. \quad (36)$$

This bundle admits local trivializations over charts $(U_\alpha, \varphi_\alpha)$ covering $\mathcal{M}_\tau$, with transition functions valued in $U(1)$. The dual interpretation of $U(1)$ as both temporal generator and gauge symmetry provides the foundational mechanism by which internal temporal structure encodes observable particle properties through the traceless internal sector, providing the geometry that underlies the gauge theory of electromagnetism and hypercharge.

TM interprets $U(1)$ as a symmetry fundamentally arising from temporal geometry. Kaluza–Klein (KK) theory [21–23] interprets charge as momentum along spatial compactified dimensions, whereas TM locates this momentum along compact temporal directions, providing a winding in internal time. This unifies quantum evolution and gauge interaction under a shared temporal symmetry [23], where geometric quantization formalism identifies such phase evolution with holonomy in fiber bundles and physical observables emerge from winding numbers [17].

IV. $U(3)$

The internal temporal sector is modeled by a rank-3 Hermitian bundle $E \to \mathcal{M}_\tau$, whose unitary automorphisms define a $U(3)$ structure. Its determinant line carries the abelian phase, while its traceless sector carries nonabelian internal structure. A temporal manifold with higher-rank unitary symmetry extends the geometric description of gauge symmetries.

The Lie group $U(n)$ is central to gauge theories and field interactions in quantum mechanics and particle physics [12, 13], where n reflects the number of coupled internal phase modes providing for the emergence of mass, flavor, and gauge.

Defining the Lie group as

$$U(n) = \left\{U \in \mathbb{C}^{n\times n} \mid U^\dagger U = I\right\}, \tag{37}$$

and choosing a basis of Hermitian matrices $T^A = (T^A)^\dagger$ $(A = 0, \dots, n^2 - 1)$, the corresponding Lie algebra elements are anti-Hermitian and spanned by iT^A, in the physics' convention. Any $X \in \mathfrak{u}(n)$ can be written as $X = i\,\alpha_A T^A$ with real coefficients $\alpha_A \in \mathbb{R}$. The Lie algebra is

$$\mathfrak{u}(n) = \{X \in M_n(\mathbb{C}) \mid X^\dagger = -X\}. \tag{38}$$

For the real vector space of Hermitian generators under gauge transformation with Lie bracket induced by the commutator $[X, Y] = XY - YX$, an n component complex matter field transforms as

$$\psi(x) \mapsto U(x)\psi(x), \quad U(x) \in U(n). \tag{39}$$

$\psi(x)$ is a complex vector valued field and transforms under the fundamental representation of $U(n)$.

The covariant derivative is

$$D_\mu = \partial_\mu + iq_n A_\mu, \tag{40}$$

where

$$A_\mu = A^\alpha_\mu T^\alpha \tag{41}$$

and the gauge symmetry is

$$A_\mu \to U A_\mu U^{-1} - \frac{i}{q_n}(\partial_\mu U)U^{-1}. \tag{42}$$

The conserved current is

$$J^{\mu\alpha} = q_n\,\bar\psi\gamma^\mu T^\alpha \psi \tag{43}$$

and the field strength is

$$\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + iq_n[A_\mu, A_\nu]. \tag{44}$$

The commutator of the covariant derivative is

$$[D_\mu, D_\nu] = iq_n\mathcal{F}_{\mu\nu}. \tag{45}$$

The group $U(n)$ has n^2 real parameters and the determinant of a unitary matrix lies on the circle with the map, $\det : U(n) \longrightarrow U(1)$ and is a group homomorphism. The higher order rotational groups $U(n)$ are isomorphic to

$$U(n) = \frac{SU(n)\times U(1)}{\mathbb{Z}_n}, \tag{46}$$

where $\mathbb{Z}_n = \{e^{2\pi i k/n}\mathbf{1} \mid k = 0, \dots, n-1\}$. This decomposition of the unitary group is standard in gauge theory [24]. $SU(n)$ denotes the special unitary group of unitary matrices with determinant 1. $\mathbb{Z}_n$ is the discrete central subgroup of $SU(n) \times U(1)$ that is identified in the quotient to produce $U(n)$, which always includes a global phase with a real cover map and the gauge connection to $SU(n)$.

In the decomposition, $\mathfrak{u}(n) = \mathfrak{su}(n) \oplus \mathfrak{u}(1)$ is the central $U(1)$. The generators T^α with $\alpha = 0, 1, \dots, n^2 - 1$ have $T^0 \propto \mathbf{1}$. For the $SU(n)$ sector, the generators T^a are traceless. The normalization is

$$\mathrm{Tr}(T^\alpha T^\beta) = \tfrac{1}{2}\delta^{\alpha\beta}. \tag{47}$$

Using the Lie algebra structure constants $f^{\alpha\beta\gamma}$, the general generator commutator is

$$[T^\alpha, T^\beta] = i\,f^{\alpha\beta\gamma}T^\gamma. \tag{48}$$

The $U(1)$ sector is $f^{0\beta\gamma} = 0, \quad [T^0, T^\alpha] = 0$. In the $SU(n)$ sector, $\{T^\alpha, T^\beta\} = \frac{1}{n}\delta^{\alpha\beta}\,\mathbf{1} + d^{\alpha\beta\gamma}T^\gamma$. The field is then

$$\mathcal{F}_{\mu\nu} = \Big(\partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + q_n\, f^{\alpha\beta\gamma}A^\beta_\mu A^\gamma_\nu\Big)T^\alpha. \tag{49}$$

Take $T^0 = \mathbf{1}_3/\sqrt{6}$ for the central $U(1)$, and the Gell-Mann matrices are $T^\alpha = \lambda^\alpha/2$ for $\alpha = 1, \dots, 8$. In the $SU(3)$ group $T^\alpha = \lambda^\alpha/2$ and $f^{123} = 1$, $f^{147} = f^{246} = f^{257} = f^{345} = \frac{1}{2}$, $f^{156} = f^{367} = -\frac{1}{2}$, $f^{458} = f^{678} = \frac{\sqrt{3}}{2}$. In the $SU(2)$ group, $T^\alpha = \sigma^\alpha/2$ and $f^{\alpha\beta\gamma} = \varepsilon^{\alpha\beta\gamma}$ [25].

$U(1)$ governs global phase and $SU(n)$ introduces internal degrees of freedom including gauge interactions. As with $U(1)$, the group $U(n)$ defines a principal fiber bundle over $\mathcal{M}_\tau$, where curvature forms encode generalized gauge field strengths,

$$F = \mathrm{d}A + i\,A \wedge A \in \Omega^2(\mathcal{M}_\tau, \mathfrak{u}(n)). \tag{50}$$

The higher gauge groups of $U(n)$ on $\mathcal{M}_\tau$ would admit additional topological invariants through higher Chern characters. These structures are defined by Chern–Weil theory, where characteristic classes arise as topological invariants associated to the curvature form of a principal bundle [26–28].

Let $\mathcal{C} : [0, 1] \to \mathcal{M}_\tau$ be a closed loop, and A connection on the $U(3)$ bundle, where the Wilson line and the internal spinors transform under flavor mixing with

$$W(\mathcal{C}) = \mathcal{P}\exp\left(-i\oint_{\mathcal{C}} A\right), \quad \chi_n \mapsto \hat W_{nk}\chi_k. \tag{51}$$

$\mathcal{P}$ denotes path ordering, and the $U(n)$ valued connection A does not commute at different points along the loop. The Wilson line $W(\mathcal{C})$ is the path ordered exponential solving $\frac{d}{ds}W(s) = -iA(s)W(s), \quad W(0) = \mathbf{1}, \quad W(\mathcal{C}) = W(1)$. χ transforms in the fundamental representation of $U(n)$ and $W(\mathcal{C})$ is an $n \times n$ unitary matrix obtained by parallel transport along $\mathcal{C}$.

V. CONJUGATE TEMPORAL WINDINGS

The temporal manifold in TM is not modeled only by its tangent directions, but by the generalized bundle

$$\mathcal{E}_\tau := T\mathcal{M}_\tau \oplus T^*\mathcal{M}_\tau, \tag{52}$$

whose fibers carry the canonical split-signature pairing

$$\langle X+\xi,\ Y+\eta\rangle = \frac{1}{2}\big(\xi(Y)+\eta(X)\big), \qquad X,Y \in T\mathcal{M}_\tau,\ \xi,\eta \in T^*\mathcal{M}_\tau. \tag{53}$$

For $\dim \mathcal{M}_\tau = 3$, this pairing has signature $(3,3)$, so the natural structure group of $\mathcal{E}_\tau$ is $O(3,3)$, or $SO(3,3)$ after fixing orientation.

The use of $SO(3,3)$ reflects a basic feature of TM where temporal geometry comes with two mutually paired directions at each point from the phase-flow directions in $T\mathcal{M}_\tau$ and conjugate phase-gradient directions in $T^*\mathcal{M}_\tau$. In this sense the temporal sector already contains a dual winding, providing a geometric origin of the two global windings.

To pass from the real split-signature bundle to internal unitary structure, TM chooses a generalized metric together with a compatible generalized complex structure on $\mathcal{E}_\tau$ [29–31]. After complexification, this produces a decomposition

$$(\mathcal{E}_\tau)_{\mathbb{C}} = \mathcal{W} \oplus \overline{\mathcal{W}}, \tag{54}$$

where $\mathcal{W}$ is the $+i$ eigenbundle and $\overline{\mathcal{W}}$ is its complex conjugate $-i$ eigenbundle. Each has complex rank 3, and together they define a $U(3)$ reduction of the internal generalized structure.

The pair $(\mathcal{W}, \overline{\mathcal{W}})$ is the mathematical expression of the two global temporal windings in TM which are conjugate realizations of the same underlying temporal holonomy. For any closed loop $\mathcal{C} \subset \mathcal{M}_\tau$, parallel transport gives unitary holonomies

$$W_{\mathcal{W}}(\mathcal{C}) \in U(3), \qquad W_{\overline{\mathcal{W}}}(\mathcal{C}) = \overline{W_{\mathcal{W}}(\mathcal{C})}. \tag{55}$$

Passing to determinant lines,

$$L_{\mathcal{W}} := \det \mathcal{W},\ L_{\overline{\mathcal{W}}} := \det \overline{\mathcal{W}}, \tag{56}$$

the Hermitian structure identifies

$$L_{\overline{\mathcal{W}}} \cong \overline{L_{\mathcal{W}}} \cong L^*_{\mathcal{W}}. \tag{57}$$

Hence the abelian holonomies are conjugate inverse phases:

$$\det W_{\mathcal{W}}(\mathcal{C}) = e^{-i\vartheta(\mathcal{C})},\ \det W_{\overline{\mathcal{W}}}(\mathcal{C}) = e^{+i\vartheta(\mathcal{C})}. \tag{58}$$

This is the precise sense in which TM has two global windings. One winding carries the phase $e^{-i\vartheta}$ and the other carries the conjugate winding $e^{+i\vartheta}$. They are opposite orientations of one and the same temporal circle, and equivalently two conjugate lifts of a single global temporal $U(1)$.

TM then makes the physical assignment

$$E := \mathcal{W},\ C := \overline{\mathcal{W}}, \tag{59}$$

where E is the electroweak-Higgs bundle and C is the color bundle. This assignment is a structural ansatz where the two conjugate temporal windings are taken to carry the two complementary nonabelian sectors of the Standard Model.

On the electroweak side, choosing a holonomy-preserved splitting

$$E = V \oplus L_0,\ \mathrm{rank}(V) = 2,\ \mathrm{rank}(L_0) = 1. \tag{60}$$

This reduces the traceless part of $U(3)$ on E to the subgroup

$$SU(2)_L \times U(1)_Y \subset SU(3)_E, \tag{61}$$

where $SU(2)_L$ acts on V and $U(1)_Y$ is generated by the traceless diagonal direction orthogonal to the locked determinant phase. The off-diagonal coset directions

$$\mathrm{Hom}(L_0, V) \subset \mathrm{End}(E) \tag{62}$$

transform as an $SU(2)_L$ doublet with hypercharge $+\frac{1}{2}$, and therefore furnish the Higgs-like sector.

On the conjugate side, the bundle

$$C = \overline{\mathcal{W}} \tag{63}$$

retains the full traceless unitary algebra

$$\mathfrak{su}(3)_C \subset \mathfrak{u}(3)_C, \tag{64}$$

which TM identifies with the color gauge sector. In this way the conjugate winding carries the $SU(3)_C$ structure, while the original winding carries the electroweak-Higgs reduction.

Because these gauge actions live on different conjugate sectors, they commute. The internal gauge content is therefore the direct product

$$G_{\mathrm{SM}} = SU(3)_C \times SU(2)_L \times U(1)_Y. \tag{65}$$

The determinant $U(1)$ of the two sectors is the same temporal phase seen with opposite orientation in the conjugate windings. This common determinant line is phase-locked to the external $\mathrm{Spin}^c(1,3)$ line, so that the abelian temporal holonomy supplies proper time.

The role of $SO(3,3)$ is more than kinematic. It supplies the generalized bundle whose compatible unitary reduction produces a conjugate pair of global temporal windings. One member of the pair carries the Higgs-electroweak structure, the other carries color. Together they realize the complete Standard Model product gauge group as two complementary aspects of a single temporal geometry.

VI. PHASE LOCKING

Definition: The phase lock is a chosen isomorphism of Hermitian line bundles with connection on $\mathcal{M}_7$,

$$\Lambda:\ (\pi_\tau^* L, \pi_\tau^* \mathcal{A}) \xrightarrow{\sim} (\pi_4^* L_{\mathrm{Spin}^c}, \pi_4^* A_{\mathrm{Spin}^c}), \tag{66}$$

together with the induced dual isomorphism

$$\Lambda^*:\ (\pi_\tau^* L^*, \pi_\tau^* \mathcal{A}^*) \xrightarrow{\sim} (\pi_4^* L^*_{\mathrm{Spin}^c}, -\pi_4^* A_{\mathrm{Spin}^c}).$$

Consequently, the Chern classes

$$\begin{aligned} c_1(\pi_\tau^* L) &= c_1(\pi_4^* L_{\mathrm{Spin}^c}), \\ c_1(\pi_\tau^* L^*) &= -\, c_1(\pi_4^* L_{\mathrm{Spin}^c}), \end{aligned} \tag{67}$$

and the curvatures satisfy

$$\begin{aligned} \pi_\tau^* F_{\det} &= \pi_4^* F_{\mathrm{Spin}^c}, \\ \pi_\tau^* F^*_{\det} &= -\, \pi_4^* F_{\mathrm{Spin}^c}. \end{aligned} \tag{68}$$

The lifted phase of the locked line then provides the external proper-time parameter from a single global temporal $U(1)$. The phase lock is the chosen identification of the determinant line of the internal $U(3)$ bundle with the auxiliary line of the external $\mathrm{Spin}^c(1,3)$ structure.

Let $L_{\mathrm{lock}} \to \mathcal{M}_7$ denote the distinguished principal $U(1)$ bundle carrying the universal time phase. Its holonomies are generated by $W(\mathcal{C})$. The lifted phase $\tilde{\theta} \in \mathbb{R}$ defines a covering map $\mathrm{Cov}: \mathbb{R} \to U(1)$ with $\mathrm{Cov}(\tilde{\theta}) = e^{-i\tilde{\theta}}$ and winding number $w = \dfrac{1}{2\pi}\Delta\tilde{\theta}$. The unwrapped temporal parameter t in the external Lorentz sector is identified with $\tilde{\theta}$ and the phase lock.

The diagonal phase lock identifies the universal cover of the locked $U(1)$ phase with the external proper-time coordinate. Let A_{lock} denote a local representative of the locked $U(1)$ connection on L_{lock}. In adapted gauges the lock may be represented by

$$A_{\mathrm{lock}} \sim \pi_\tau^* \mathcal{A} \sim \pi_4^* A_{\mathrm{Spin}^c}, \tag{69}$$

with locally defined curvature

$$F_{\mathrm{lock}} = dA_{\mathrm{lock}}, \tag{70}$$

while globally the corresponding curvature is the common 2-form determined by the locked Chern class. The external time direction is not an additional independent coordinate. It is the lifted determinant phase of the internal $U(3)$ structure, and $e^m{}_\mu$ and $g_{\mu\nu}(\phi)$ respond to internal energy through the locked Spin^c line.

In the $U(3)$ reduction, $U(3)$ acts on the rank-3 Hermitian bundle $\mathcal{E} \to \mathcal{M}_\tau$ with connection $A \in \Omega^1(\mathcal{M}_\tau, \mathfrak{u}(3))$. The standard isogeny of $U(3) = \frac{SU(3)\times U(1)}{\mathbb{Z}_3}$ is

$$A \;=\; A^{traceless} + \frac{1}{3}\,\mathrm{tr}(A)\,\mathbf{1}. \tag{71}$$

The internal decomposition

$$1 \to SU(3) \to U(3) \xrightarrow{\det} U(1) \to 1 \tag{72}$$

identifies a traceless $SU(3)$ sector as the $SU(3)$ fiber base and a determinant $U(1)$ sector attachment, which is phase-locked to the external Lorentzian $\mathrm{Spin}^c(1,3)$ line.

This identifies the total internal temporal phase which is the sum of internal eigenphases in $U(3)$ with the external proper time $U(1)$ phase governing Spin^c transport on $\mathcal{M}_4$. There is a single global temporal $U(1)$. The temporal evolution on the physical subspace $\mathcal{H}_\tau$ is a pure fiber rotation,

$$\hat{U}_\tau(t) = e^{-iE_\tau t}\mathbf{1}_{\mathcal{H}_\tau}, \qquad -i\hat{H}_\tau \in \mathfrak{u}(1), \tag{73}$$

while in general $-i\hat{H} \in \mathfrak{u}(n)$ with $\hat{U}(t) = e^{-i\hat{H}t} \in U(n)$.

The phase lock induces the external Lorentzian sector through the chronometric field, $\phi: T\mathcal{M}_4 \to \mathbb{R}$, homogeneous quadratic function on the tangent space $T_p\mathcal{M}_4$, such that $\phi_p(v) := -g_p(v,v)$ has a Lorentzian signature $(1,3)$. The metric is reconstructed by polarization,

$$g_p(u,v) = -\frac{1}{2}\big[\phi_p(u+v) - \phi_p(u) - \phi_p(v)\big]. \tag{74}$$

Thus $(\mathcal{M}_4, g(\phi))$ carries the emergent orthonormal structure group $SO(1,3)$. The external Lorentzian metric is reconstructed from the chronometric field ϕ, so that $(\mathcal{M}_4, g(\phi))$ carries proper time along a clock's worldline [1, 3, 32],

$$d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu, \tag{75}$$

where operational time measured by clocks in the emergent external spacetime is the proper time induced by the phase-locked metric.

VII. TEMPORAL MANIFOLD

$$\begin{array}{ccc} \mathcal{G} = U(3)\times_{U(1)} \mathrm{Spin}^c(1,3) & \longrightarrow & U(3) \\ \downarrow & & \downarrow \det \\ \mathrm{Spin}^c(1,3) & \xrightarrow{\zeta} & U(1) \end{array} \tag{76}$$

Locally working in a product chart $\mathcal{M}_7 = \mathcal{M}_4 \times \mathcal{M}_\tau$, the associated line bundle is the locked $U(1)$ line $L_{\mathrm{lock}} \to \mathcal{M}_7$ which may be topologically nontrivial. The phase lock identifies the external time parameter with the lifted determinant phase.

Definition: Let $\mathcal{M}_7$ be a seven-dimensional manifold equipped with a principal structure, $\mathcal{G} = U(3) \times_{U(1)} \mathrm{Spin}^c(1,3)$. Locally one may use adapted coordinates (x^μ, θ^a) for the external and internal sectors. The temporal manifold is equipped with an atlas $\{(U_\alpha, \varphi_\alpha)\}$, which enables the consistent definition of gauge fields, spinor bundles, and holonomies essential for encoding the SM gauge symmetries. $\varphi_\alpha: U_\alpha \to \mathbb{R}^3$ are smooth coordinate charts ensuring compatibility on overlaps $U_\alpha \cap U_\beta$. These charts allow for local trivializations of the fiber bundle, permitting local sections and connections that transform under the group $U(3)$. This manifold serves as the base of a principal bundle,

$$U(3) \hookrightarrow P \xrightarrow{\pi} \mathcal{M}_\tau, \quad A \in \Omega^1(P, \mathfrak{u}(3)), \tag{77}$$

where P is the total space, π the projection, and A the connection 1-form.

The curvature 2-form encodes how the temporal geometry twists across local trivializations and the bundle topology is classified by its characteristic Chern classes which distinguishes physically distinct sectors of field configurations. Topologically, $\mathcal{M}_\tau$ is compact, orientable, and admits nontrivial cohomology groups $H^2(\mathcal{M}_\tau, \mathbb{Z})$, relevant for quantized holonomies. While nothing requires $\mathcal{M}_\tau$ to be globally trivial, local trivializations exist over each chart domain, supporting smooth gluing using the group $U(3)$.

The connections determine the manifold structure. Given (A_C, A_L, A_Y), the internal curvatures (F_C, F_L, F_Y) fixes the internal energy density and hence the total temporal energy E_τ. Through the lock and the first-order equations, this energy determines the external time direction and sources the tetrad and metric from

$$G_{\mu\nu}[g(\phi)] \;=\; \kappa\, T_{\mu\nu}[F_C, F_L, F_Y, g, \chi]. \tag{78}$$

The choice of internal connection and its holonomy class selects the external manifold structure dynamically making the forbidden mixed and drifting components

$$\begin{gathered} g_{\mu a} = 0, \qquad \partial_\mu g_{ab} = 0, \qquad \partial_\mu A_a = 0, \\ \Downarrow \\ \sqrt{|g^{(7)}|} = \sqrt{-g^{(4)}}\sqrt{g^{(\tau)}}, \\ R[g^{(7)}] = R[g^{(4)}] + R[g^{(\tau)}]. \end{gathered} \tag{79}$$

The internal metric $g^{(\tau)}$ is rigid over $\mathcal{M}_4$, and the locked $U(1)$ has only external curvature F_{Spin}, where the internal $U(3)$ connection $A \in \Omega^1(\mathcal{M}_\tau, \mathfrak{u}(3))$ has curvature $F = dA + iA \wedge A$, with the determinant part locked. External spinors live in the $\text{Spin}^c(1,3)$ bundle, where internal spinors are sections over $\mathcal{M}_\tau$.

In order to formulate the action from the product manifold $\mathcal{M}_7$, assume a block-diagonal product metric

$$g^{(7)} = g^{(4)} \oplus g^{(\tau)}, \tag{80}$$

so that in coordinates

$$ds^2 = g^{(4)}_{\mu\nu}(x)\, dx^\mu dx^\nu + g^{(\tau)}_{ab}(\theta)\, d\theta^a d\theta^b, \tag{81}$$

and the volumes can be defined by

$$\kappa_4 := \int \sqrt{|g^{(4)}|} d^4x, \quad \kappa_\tau := \int \sqrt{|g^{(\tau)}|} d^3\theta. \tag{82}$$

Assume the total Lagrangian has the form

$$\mathcal{L} = R + \mathcal{L}_{\mathcal{S}}, \tag{83}$$

with a source contribution that also separates,

$$\mathcal{L}_S = \mathcal{L}_{S,4} + \mathcal{L}_{S,\tau}. \tag{84}$$

Then

$$\mathcal{L} = (R_4 + \mathcal{L}_{S,4}) + (R_\tau + \mathcal{L}_{S,\tau}). \tag{85}$$

Define the submanifold Lagrangians

$$\mathcal{L}_4 := R_4 + \mathcal{L}_{S,4}, \quad \mathcal{L}_\tau := R_\tau + \mathcal{L}_{S,\tau}, \tag{86}$$

so that

$$\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_\tau. \tag{87}$$

The total action on the product manifold is

$$S = \int d^7x \sqrt{|g^{(7)}|}\, \mathcal{L}. \tag{88}$$

Because $\mathcal{L}_4$ depends only on x and $\mathcal{L}_\tau$ depends only on θ, Fubini's theorem gives

$$\begin{aligned} S = \int d^7x \sqrt{|g^{(7)}|}\mathcal{L}_4(x) \,&+ \int d^7x \sqrt{|g^{(7)}|}\mathcal{L}_\tau(y) \\ &= \int d^4x \sqrt{|g^{(4)}|}\mathcal{L}_4 \int d^3\theta \sqrt{|g^{(\tau)}|} \\ &+ \int d^4x \sqrt{|g^{(4)}|} \int d^3\theta \sqrt{|g^{(\tau)}|}\mathcal{L}_\tau \\ = \kappa_\tau \int d^4x \sqrt{|g^{(4)}|}\mathcal{L}_4 \,&+ \kappa_4 \int d^3\theta \sqrt{|g^{(\tau)}|}\mathcal{L}_\tau\,. \end{aligned} \tag{89}$$

Substituting the Ricci-plus-source split yields

$$\begin{aligned} S = \kappa_\tau \int d^4x \sqrt{|g^{(4)}|}(R_1 + \mathcal{L}_{S,1}) \\ + \kappa_4 \int d^3\theta \sqrt{|g^{(\tau)}|}(R_2 + \mathcal{L}_{S,2})\,. \end{aligned} \tag{90}$$

If the Ricci terms are written with Einstein-Hilbert normalization, then

$$\mathcal{L}_4 = \frac{1}{16\pi G_4} R_4 + \mathcal{L}_{S,4}, \;\; \mathcal{L}_\tau = \frac{1}{16\pi G_\tau} R_\tau + \mathcal{L}_{S,\tau}. \tag{91}$$

and

$$\begin{aligned} S = \kappa_\tau \int d^4x \sqrt{|g^{(4)}|}\left(\frac{R_4}{16\pi G_4} + \mathcal{L}_{S,4}\right) \\ + \kappa_4 \int d^3\theta \sqrt{|g^{(\tau)}|}\left(\frac{R_\tau}{16\pi G_\tau} + \mathcal{L}_{S,\tau}\right). \end{aligned} \tag{92}$$

In a decoupled zeroth-order theory, it is assumed that the temporal manifold is rigid and the Lorentzian manifold is emergent by $\mathcal{L}_{S,1} = 0$ and $R_\tau = 0$. The submanifold Lagrangians then reduce to $\mathcal{L}_4 = R_4$, and $\mathcal{L}_\tau = \mathcal{L}_{S,\tau}$, and the total Lagrangian becomes $\mathcal{L}^{(0)} = R_4 + \mathcal{L}_{S,\tau}$. Accordingly, the action simplifies to

$$S^{(0)} = \kappa_\tau \int d^4x \sqrt{|g^{(4)}|}R_4 \,+ \kappa_4 \int d^3\theta \sqrt{|g^{(\tau)}|}\mathcal{L}_{S,2}\,. \tag{93}$$

With Einstein-Hilbert normalization this is

$$S^{(0)} = \frac{\kappa_\tau}{16\pi G_4} \int d^4x \sqrt{|g^{(4)}|} R_4 + \kappa_4 \int d^3\theta \sqrt{|g^{(\tau)}|} \mathcal{L}_{S,2} \,. \tag{94}$$

This is the decoupled or zeroth-order theory. It consists of a purely geometric sector on $\mathcal{M}_4$, and a purely source sector on $\mathcal{M}_\tau$.

Thus the zeroth-order solution is a direct product of an M_1 vacuum geometry and an M_2 source configuration and the energy-momentum tensor from Einstein's field equations is then

$$\frac{\delta S}{\delta g^{\mu\nu}} = \int d^4x \sqrt{|g^{(4)}|} \left(\frac{\kappa_\tau}{16\pi G_4} G_{\mu\nu} - \frac{1}{2} T_{\mu\nu}, \right), \tag{95}$$

where

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g^{(4)}}} \frac{\delta}{\delta g^{\mu\nu}} \int d^4x \sqrt{-g^{(4)}} \, \mathcal{L}_\tau. \tag{96}$$

The full coupled theory can now be organized as a perturbation about $S^{(0)}$. Introduce a perturbation parameter ϵ and write

$$S = S^{(0)} + \epsilon S^{(1)} + \epsilon^2 S^{(2)} + \cdots \,. \tag{97}$$

A convenient first-order decomposition is

$$\mathcal{L} = \mathcal{L}^{(0)} + \epsilon \mathcal{L}_{\text{int}}, \tag{98}$$

with

$$\mathcal{L}_{\text{int}} = \mathcal{L}_{S,4} + R_\tau + \mathcal{L}_{\text{mix}}. \tag{99}$$

Here $\mathcal{L}_{S,4}$ restores source structure on M_4, R_τ restores curvature on M_τ, and $\mathcal{L}_{\text{mix}}$ denotes a genuinely mixed interaction term that cannot be written as a pure x-term plus a pure θ-term.

Thus the full Lagrangian becomes

$$\mathcal{L} = R_4 + \mathcal{L}_{S,\tau} + \epsilon \big(\mathcal{L}_{S,4} + R_\tau + \mathcal{L}_{\text{mix}} \big). \tag{100}$$

The corresponding first-order action correction is

$$S^{(1)} = \kappa_\tau \int d^4x \sqrt{|g^{(4)}|} \mathcal{L}_{S,4} + \kappa_4 \int d^3\theta \sqrt{|g^{(\tau)}|} R_\tau + \int d^7x \sqrt{|g^{(7)}|} \mathcal{L}_{\text{mix}}. \tag{101}$$

First principles construction of the action using universal symmetry principles provides external diffeomorphism invariance and second order metric dynamics singling out the Einstein–Hilbert term on $\mathcal{M}_4$, using internal gauge invariance adjoint $U(3)$ with F^2 and topological Chern–Simons term. Spin geometry and minimal coupling single out the Dirac actions external and internal with covariant derivatives built from the corresponding connections. The phase lock fixes the universal time eliminating an abelian redundancy. It enforces the locked rigid ansatz, yielding constant κ and stable gauge couplings, and it pushes all nontrivial internal physics into the traceless $SU(3)$ sector, where holonomy generate the discrete internal spectrum responsible for masses.

VIII. THE STANDARD MODEL

In the same way that SO(10) grand unifying theories arrange quarks and leptons into a single 16-dimensional spinor representation [33, 34], TM arranges Lorentz spin and flavor into a single $U(3) \times_{U(1)} \text{Spin}^c(1,3)$ spinor bundle, with internal quantum numbers arising from holonomy in the temporal manifold. The temporal $U(1)$ appearing after locking is the central phase of the full unitary evolution promoted to a geometric holonomy and identified with the internal determinant phase. The internal geometry contains two conjugate rank-3 sectors identifing one with the color bundle C and the other with the electroweak-Higgs bundle E. The electroweak subgroup is embedded in the traceless part of $U(3)$ acting on E, while the full traceless part of the conjugate sector furnishes $SU(3)_C$.

In TM the electroweak embedding is taken inside the traceless sector of $U(3)$ acting on $\mathcal{E}$. After locking, the trace $U(1)$ is fixed, where the dynamical internal gauge directions lie in $\mathfrak{su}(3)$. From Equation (46) the residual gauge group is $SU(3)$. The chosen holonomy preserved rank-2 subbundle $V \subset E$, where $E = V \oplus L_0$ and $\det E = \det V \otimes L_0$. This embeds $SU(2)_L$ as endomorphisms of V and picks a traceless diagonal $U(1)_Y$ orthogonal to both T_3 and the locked trace.

In the $SU(3)$ embedding $SU(2)_L \times U(1)_Y \subset SU(3)$, $SU(2)_L$ acts on the first two components generated by the Gell-Mann matrices $\lambda_1, \lambda_2, \lambda_3$ in the chosen upper-left 2×2 block as $T^a_L = \frac{\lambda_a}{2}, \quad a = 1,2,3$. Taking the hypercharge as the diagonal generator in $SU(3)$ as $Y = y \, \text{diag}(1,1,-2)$, where y is a normalization constant. Then picking one $SU(2)$, the first two components form an $SU(2)$ doublet and the third component is an $SU(2)$ singlet with hypercharge eigenvalues $Y_1 = Y_2 = y$ and $Y_3 = -2y$. Then the coset

$$\Pi = \frac{SU(3)}{SU(2)_L \times U(1)_Y} \tag{102}$$

is represented at the Lie algebra level by the generators that are not in $\mathfrak{su}(2)_L$ or Y. The adjoint of $SU(3)$ has 8 generators and 3 of them are $SU(2)_L(\lambda_1, \lambda_2, \lambda_3)$. The one $U(1)_Y$ direction is identified as λ_8. The remaining generators that span the coset are $\lambda_4, \lambda_5, \lambda_6, \lambda_7$. The complex doublet can then be written as

$$\begin{aligned} E_{13} &\sim \lambda_4 + i\lambda_5, \\ E_{23} &\sim \lambda_6 + i\lambda_7, \end{aligned} \tag{103}$$

which rotates under $SU(2)_L(E_{13}, E_{23})$.

In order to compute the hypercharge, consider E_{13}, which mixes $(1,3)$ and E_{23} which mixes $(2,3)$. Acting on the basis $\{e_1, e_2, e_3\}$, E_{13} takes $e_3 \mapsto e_1$ under Y, the phase of e_i is Y_i where $Y(e_1) = y$, $Y(e_2) = y$, $Y(e_3) = -2y$. Then the charge of the raising operator E_{13} is $Y(E_{13}) = Y(e_1) - Y(e_3) = y - (-2y) = 3y$ and $Y(E_{23}) = Y(e_2) - Y(e_3) = y - (-2y) = 3y$, where the coset doublet (E_{13}, E_{23}) has the hypercharge $Y = 3y$.

Identifying the Higgs doublet's quantum number as $Y_H = +\frac{1}{2}$ then $3y = \frac{1}{2} \Rightarrow y = \frac{1}{6}$ giving the expected hypercharge. The Higgs doublet field $\xi(x)$ arises from fluctuations of the internal connection along the coset directions spanned by (E_{13}, E_{23}). This uniquely fixes the embedding to $Y = \frac{1}{6}\text{diag}(1, 1, -2)$, demonstrating that TM produces the correct hypercharge normalization with $y = 1/6$. The fundamental 3 of $SU(3)$ decomposes under $SU(2)_L \times U(1)_Y$ as the two components with $Y = +1/6$ and a component with $Y = -1/3$, where $\mathbf{3} \to \mathbf{2}_{+1/6} \oplus \mathbf{1}_{-1/3}$.

After locking, the holonomy-preserved splitting of the rank-3 internal bundle

$$E = V \oplus L_0, \tag{104}$$

where $\text{rank}(V) = 2$ and $L_0 := V^\perp$ is a line bundle. Under the Hermitian traceless diagonal generator

$$Y_0 = \frac{1}{6}\text{diag}(1, 1, -2), \tag{105}$$

the two summands transform as

$$V \cong \mathbf{2}_{+1/6}, \qquad L_0 \cong \mathbf{1}_{-1/3}. \tag{106}$$

The coset directions are the off-diagonal maps between these summands,

$$H \cong \text{Hom}(L_0, V) \cong V \otimes L_0^*, \tag{107}$$

so the Higgs bundle carries

$$Y(H) = Y(V) - Y(L_0) = \frac{1}{6} - (-\frac{1}{3}) = +\frac{1}{2}. \tag{108}$$

Thus, the coset mode is an $SU(2)_L$ doublet with the Standard Model Higgs hypercharge.

The lepton doublet and the right-handed charged lepton arise from the same structure once the unique locked temporal line $L = \det(E)$ is used to twist the base $SU(3)$ weights. Normalizing the temporal line so that tensoring by L^q shifts the effective hypercharge by $q/3$, where $Y_{\text{eff}} = Y_0 + \frac{q}{3}$. Since $V^* \cong \mathbf{2}_{-1/6}$, one obtains

$$L_L \cong V^* \otimes L^{-1} \cong \mathbf{2}_{-1/2}, \quad e_R \cong L_0 \otimes L^{-2} \cong \mathbf{1}_{-1}, \tag{109}$$

and similarly

$$\begin{gathered} Q_L \cong C \otimes V,\ d_R \cong C \otimes L_0, \\ u_R \cong C \otimes L_0^* \otimes L,\ \nu_R \cong L_0 \otimes L. \end{gathered} \tag{110}$$

Here C denotes the color triplet bundle. In this way the full one-generation hypercharge pattern is produced by the traceless $SU(3)$ embedding together with integer powers of the same locked temporal line.

The construction above reproduces the one-generation Standard Model hypercharges listed in Table I. The three generations are then introduced by assigning the three low-lying temporal sectors to three copies of these bundle types. It provides a common geometric origin for the electroweak embedding, the Higgs doublet, and the associated abelian charge assignments.

Field f	Bundle	$(SU(3)_C, SU(2)_L)_Y^{T^3}$	Charge Q
(u_L, d_L)	$C \otimes V$	$(\mathbf{3}, \mathbf{2})_{+1/6}^{\pm 1/2}$	$(+\frac{2}{3}, -\frac{1}{3})$
u_R	$C \otimes L_0^* \otimes L$	$(\mathbf{3}, \mathbf{1})_{+2/3}^{0}$	$+\frac{2}{3}$
d_R	$C \otimes L_0$	$(\mathbf{3}, \mathbf{1})_{-1/3}^{0}$	$-\frac{1}{3}$
(ν_L, e_L)	$V^* \otimes L^{-1}$	$(\mathbf{1}, \mathbf{2})_{-1/2}^{\pm 1/2}$	$(0, -1)$
e_R	$L_0 \otimes L^{-2}$	$(\mathbf{1}, \mathbf{1})_{-1}^{0}$	-1
ν_R	$L_0 \otimes L$	$(\mathbf{1}, \mathbf{1})_{0}^{0}$	0

TABLE I. One-generation bundle assignment in TM. The rank-2 bundle V and line bundle L_0 arise from the decomposition $E = V \oplus L_0$ of the internal temporal bundle, $L = \det(E)$ is the locked temporal line, and C denotes the color triplet bundle. Each Standard Model fermion f transforms in a representation of the internal temporal group $U(3)$, whose Cartan generators T^3 and Y act on the temporal fiber with eigenvalues (t^3, y). Phase locking and the $SU(3)$ embedding identify these internal weights with the external weak isospin and hypercharge, $(T^3, Y) \equiv (t^3, y)$, so that the electric charge is $Q = T^3 + Y$.

The $U(3)$ symmetry of $\mathcal{E}$ carries both the electroweak subgroup and the coset directions that generate the Higgs doublet in the gauge–Higgs unification interpretation. Reducing the manifold space to $U(2)$ the gauge-Higgs mode disappears and only the electroweak group can exist. Temporal manifolds larger than 3 result in more Higgs doublets. Generalizing the coset to $SU(2)_L \times U(1)_Y \times SU(n-2) \subset SU(n)$, where $\dim SU(n) = n^2 - 1$, $\dim SU(2)_L = 3$, $\dim SU(n-2) = (n-2)^2 - 1$, and $\dim U(1)_Y = 1$. The subgroup $\Pi_n = SU(2)_L \times U(1)_Y \times SU(n-2)$ has $\dim \Pi_n = 3 + 1 + (n-2)^2 - 1 = (n-2)^2 + 3 = n^2 - 4n + 7$. The coset dimensions is then $\dim \dfrac{SU(n)}{SU(2)_L \times U(1)_Y \times SU(n-2)} = (n^2 - 1) - (n^2 - 4n + 7) = 4n - 8 = 4(n-2)$, where each complex $SU(2)$ doublet has 4 degrees of freedom and $n - 2$ Higgs doublets.

The discovery of a 125 GeV scalar with SM-like couplings by ATLAS and CMS, together with combined measurements of Higgs signal strengths in multiple channels, disfavors large admixtures of additional Higgs doublets or other light scalar multiplets that mix significantly with the observed 125 GeV state. Extended Higgs sectors such as two-Higgs-doublet models remain phenomenologically allowed where extra states are heavy and weakly mixed, but there is at present no positive evidence for additional doublets [35–38].

Defining each SM Weyl multiplet [39] with the field, f, enumerates the SM of particles with

$$f \in \{Q_L, u_R, d_R, L_L, e_R, \nu_R\}, \tag{111}$$

where L_L is the other doublet (ν_l, e_l). After the breaking $U(3) \to SU(2)_L \times U(1)_Y$, these internal representation generate the visible fermion multiplets. The phase lock identifies the external Spin^c $U(1)$ with the internal determinant $U(1)$ so there is a single abelian connection. The internal electroweak embedding is a choice of subgroup $U(3) \to SU(2)_L \times U(1)_Y$. Y is taken as a linear combination of a traceless Cartan generator in $su(3)$ and the locked $u(1)$ generator subject to $U(3) = (SU(3) \times U(1))/\mathbb{Z}_3$. Table I enumerates the Standard Model Weyl multiplets, where ν_R is a true singlet identified as a right sterile neutrino [40]. The entire SM structure content exist as different aspects of the same temporal manifold's $U(3)$ group. The usual SM anomaly cancellations are inherited because the one-generation bundle assignments reproduce the SM hypercharge pattern.

The electroweak gauge group arises from gauging a chiral subgroup of the temporal $U(3)$ structure. Specifically, an $SU(2) \times U(1)$ subgroup acts nontrivially on left-handed fermions, while right-handed fermions transform as singlets under $SU(2)$. The Higgs field arises from the coset directions of this reduction, and its vacuum expectation value pairs left- and right-handed modes using the temporal Dirac spectrum. Because the phase lock identifies the determinant $U(1)$ with the external Spin^c line, TM contains no independent additional abelian factor. This construction yields exactly the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, with no additional light Z' or extra $U(1)$ factors, which aligns with collider limits on light Z' sectors.

IX. TEMPORAL MECHANICS

After locking, the internal gauge dynamics reside in the traceless $\mathfrak{su}(3)$ sector, while the shared $U(1)$ governs universal time transport. Let γ^μ be a four–dimensional Clifford representation on $\mathcal{M}_4$ obeying

$$\{\gamma^\mu, \gamma^\nu\} = 2g_4^{\mu\nu}, \tag{112}$$

and define the external chirality operator

$$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3, \quad (\gamma^5)^2 = \mathbf{1}, \quad \{\gamma^5, \gamma^\mu\} = 0. \tag{113}$$

Let γ^a be a three-dimensional Euclidean Clifford representation on $\mathcal{M}_\tau$,

$$\{\gamma^a, \gamma^b\} = 2g_\tau^{ab}. \tag{114}$$

The graded product Clifford generators

$$\Gamma^\mu = \gamma^\mu \otimes \mathbf{1}, \quad \Gamma^a = \gamma^5 \otimes \gamma^a, \tag{115}$$

so that $\{\Gamma^\mu, \Gamma^\nu\} = 2g_4^{\mu\nu}$, $\{\Gamma^a, \Gamma^b\} = 2g_\tau^{ab}$, and $\{\Gamma^\mu, \Gamma^a\} = 0$. The appearance of γ^5 in Γ^a reflects the Lorentzian grading on $L^2(\mathcal{S}_{(1,3)})$.

Defining the total Dirac operator algebraically by

$$D_7 \;=\; i\Gamma^A \nabla_A \;=\; i\Gamma^\mu \nabla_\mu + i\Gamma^a \nabla_a, \tag{116}$$

then the Dirac operator on the product of the external Lorentz spin geometry with the internal temporal spectral geometry with the covariant derivatives split as

$$\nabla_\mu = \partial_\mu + \omega_\mu + i \sum_{k \in R_f} q_k \, A_\mu^{(k)}(x) \, T_k, \tag{117}$$

$$\nabla_a = \partial_a + \omega_a + iA_a(\theta), \tag{118}$$

where $A_\mu^{(k)}$ are the four–dimensional gauge fields obtained by gauging the appropriate internal symmetries over $\mathcal{M}_4$, and $A_a(\theta)$ is the rigid internal temporal connection. The field produced by the causal response to the fermion is then summed over its representation R_f. After locking, the determinant component of $A_a(\theta)$ is fixed by the shared $U(1)$ time phase, so the dynamical internal directions lie in $\mathfrak{su}(3)$.

With the representation in Equation (115), the operator splits as

$$D_7 = D_4 \otimes \mathbf{1} - \gamma^5 \otimes D_\tau, \tag{119}$$

where

$$\begin{aligned} D_4 &= i\gamma^\mu (\partial_\mu + \omega_\mu + i \sum_{k \in R_f} q_k A_\mu^{(k)}(x) T_k), \\ D_\tau &= -i\gamma^a \left(\partial_a + \omega_a + iA_a(\theta)\right). \end{aligned} \tag{120}$$

Lorentz chirality is governed by γ^5 and the projectors are

$$P_{L,R} = \tfrac{1}{2}(1 \mp \gamma^5) \tag{121}$$

on the external spinor bundle.

Assume a separated expansion

$$\Psi(x, \theta) = \sum_n \psi_n(x) \otimes \chi_n(\theta), \tag{122}$$

where the internal modes satisfy

$$D_\tau \chi_n = E_n \, \chi_n, \quad \langle \chi_n, \chi_m \rangle_\tau = \delta_{nm}. \tag{123}$$

Substitution into the Dirac equation $D_7 \Psi = 0$ and projection onto χ_n yields the effective four-dimensional equation

$$\left(D_4 + E_n \, \gamma^5\right) \psi_n(x) = 0. \tag{124}$$

E_n is the spectral scale set by the internal temporal geometry. Because $\{\gamma^5, \gamma^\mu\} = 0$, a constant chiral basis rotation converts Equation (124) into the standard massive Dirac form, so the magnitude

$$m_n := |E_n| \tag{125}$$

is the zeroth-order mass parameter associated with the n^{th} internal mode. In this chiral electroweak theory, the observed four-dimensional fermion masses arise after symmetry breaking through effective Yukawa matrices induced by the same internal geometry, where the singular values of $vY_f/\sqrt{2}$ furnish the physical masses, while m_n sets the underlying temporal scale from which those Yukawa couplings are built.

Using $\{\gamma^5, \gamma^\mu\} = 0$ and $(\gamma^5)^2 = \mathbf{1}$, one finds

$$\left(D_4 + E\gamma^5\right)\left(D_4 - E\gamma^5\right) = D_4^2 - E^2, \tag{126}$$

so the dispersion is that of a mass $|E|$. Decomposing $\psi_n = \psi_{nL} + \psi_{nR}$, pairs left- and right-handed Lorentz components through the same internal eigenvalue. This Lorentzian chirality structure is intrinsic to the $\mathrm{Spin}(1,3)$ sector.

The internal eigenvalues can be expressed as expectation values of the internal Dirac operator. With χ normalized on $\mathcal{M}_\tau$ with a flat internal frame,

$$E_\tau = \int_{\mathcal{M}_\tau} d^3\theta\, \sqrt{g_\tau}\, \chi^\dagger(\theta)\big(-i\gamma^a(\partial_a + iA_a)\big)\chi(\theta), \tag{127}$$

and the effective action is

$$\begin{aligned} S_4 = \int_{\mathcal{M}_4} d^4x\sqrt{-g_4}\bar{\psi}(x)\big[i\gamma^\mu\big(\partial_\mu + \\ i\sum_{k\in R_f} q_k A_\mu^{(k)} T_k\big) - m_n\big]\psi(x). \end{aligned} \tag{128}$$

This provides a geometric mass generation mechanism in which rest masses arise as the spectrum of the internal temporal Dirac operator. The geometry of $\mathcal{M}_\tau$, its spin structure, the locked determinant line, and the traceless $SU(3)$ holonomy together control the allowed internal modes and therefore the observable four–dimensional mass spectrum. In this way gauge interactions, mass generation, and the Lorentzian chirality are unified by a single phase-locked temporal geometry.

X. MASS

For a compact three-dimensional temporal manifold, $\mathcal{M}_\tau = \mathbb{T}^3$, the second cohomology $H^2(\mathbb{T}^3, \mathbb{Z}) = \mathbb{Z}^3$ has three independent generators represented by the three coordinate $\mathbb{T}^2$ subtori. These three primitive 2-cycles motivate a three-sector organization of the low-lying temporal holonomies. The three distinguished low-lying sectors are identified with the three observed generations. After phase locking removes the trace direction, the remaining traceless Cartan data of the internal $U(3)$ holonomy distinguish these sectors and set their zeroth-order spectral scales.

TM assigns an internal Hilbert space $\mathbb{H}_\tau^{(f)}$ of spinor sections on $\mathcal{M}_\tau$ transforming in a representation R_f of the internal $U(3)$ bundle, where f is the fermion index. The internal Dirac operator in this sector is

$$\begin{aligned} D_\tau^{(f)} &= -i\,\gamma^a\big(\partial_a + \omega_a + iA_a^{(f)}(\theta)\big), \\ A_a^{(f)} &= A_a^A(\theta)\, T_A^{(R_f)}, \end{aligned} \tag{129}$$

where $T_A^{(R_f)}$ are the generators of $\mathfrak{u}(3)$ in the representation R_f. Internal eigenmodes satisfy

$$\begin{aligned} D_\tau^{(f)}\, \chi_{n,i}^{(f)}(\theta) &= E_{n,i}^{(f)}\, \chi_{n,i}^{(f)}(\theta), \\ \langle \chi_{n,i}^{(f)}, \chi_{m,j}^{(f)} \rangle_\tau &= \delta_{nm}\delta_{ij}, \end{aligned} \tag{130}$$

and the corresponding four dimensional fermions have rest masses

$$m_{n,i}^{(f)} = \big|E_{n,i}^{(f)}\big|. \tag{131}$$

Here n is the index labeling the three generations, and i labels the species within a generation, where the spectrum of $D_\tau^{(f)}$ with different f and i sample different eigenvalues. The eigenvalues are determined by the internal geometry and the $U(3)$ representation R_f, where Table I shows the allowed temporal structures of the SM. It is convenient to arrange the charged fermions against (n, i) as

$$f = \begin{pmatrix} e & u & d \\ \mu & c & s \\ \tau & t & b \end{pmatrix}. \tag{132}$$

Charged fermions exist as the lowest twisted plane waves [41] compatible with a twisted boundary condition having stable eigenmodes of $D_\tau^{(f)}$ in each sector, which are aligned along inequivalent Cartan directions of $U(3)$.

Specializing to the semi-flat torus polarization $\mathcal{M}_\tau = \mathbb{T}^3$ that supplies the holonomy and choosing a lattice basis $\{\ell_a\}_{a=1}^3 \subset \mathbb{R}^3$ for Λ, which representing points on $\mathbb{T}^3$ by $\theta \in \mathbb{R}^3$ with the identification

$$\theta \sim \theta + 2\pi\ell_a. \tag{133}$$

Let $\{\ell^a\}$ denote the dual basis, $\ell^a \cdot \ell_b = \delta^a{}_b$. Define the 2π-periodic scalar coordinates along the cycles by

$$\begin{aligned} \vartheta^a(\theta) &= \ell^a \cdot \theta \\ &\Downarrow \\ \vartheta^a(\theta + 2\pi\ell_b) &= \vartheta^a(\theta) + 2\pi\,\delta^a{}_b. \end{aligned} \tag{134}$$

The rest masses are universal over $\mathcal{M}_4$ and the metric on $\mathbb{T}^3$ is taken to be

$$g_{ab} = g(\ell_a, \ell_b), \quad (g^{ab}) = (g_{ab})^{-1}. \tag{135}$$

For each fermion type f, the background of commuting Wilson lines are in the internal $U(3)$ bundle. In a commuting Cartan background, the connection components are taken to be constant and diagonalizable where

$$\begin{aligned} A^{(f)} &= \frac{1}{2\pi}\,\Theta_a^{(f)}\, d\vartheta^a, \\ \Theta_a^{(f)} &= \mathrm{diag}\big(\Theta_a^{(f,1)}, \Theta_a^{(f,2)}, \Theta_a^{(f,3)}\big), \end{aligned} \tag{136}$$

and the phases $\Theta_a^{(f,i)}$ are the eigenphases of the holonomy in the representation R_f around the a^{th} fundamental cycle. The Wilson line around that cycle is

$$\begin{aligned} W_a^{(f)} &= \exp\big(-i\,\Theta_a^{(f)}\big) = \\ &\mathrm{diag}\big(e^{-i\Theta_a^{(f,1)}}, e^{-i\Theta_a^{(f,2)}}, e^{-i\Theta_a^{(f,3)}}\big). \end{aligned} \tag{137}$$

The internal Dirac operator in sector f reduces to

$$D_\tau^{(f)} = -i\,\gamma^a\big(\partial_a + i\,A_a^{(f)}\big), \tag{138}$$

with $\omega_a = 0$ in this flat background. The temporal wavefunctions obey twisted boundary conditions

$$\chi^{(f,i)}(\theta + 2\pi\ell_a) = e^{-i\Theta_a^{(f,i)}}\,\chi^{(f,i)}(\theta). \tag{139}$$

For a plane-wave mode on the covering space,

$$\chi^{(f,i)}(\theta) = \exp\Big(i\,k_a^{(f,i)}\,\vartheta^a(\theta)\Big)\,u^{(f,i)}, \tag{140}$$

the boundary condition implies

$$e^{\,i2\pi k_a^{(f,i)}} = e^{-i\Theta_a^{(f,i)}} \Longleftrightarrow k_a^{(f,i)} = n_a - \frac{\Theta_a^{(f,i)}}{2\pi}, \tag{141}$$

where $n_a \in \mathbb{Z}$. The corresponding Dirac eigenvalues satisfy

$$E^2 = g^{ab}k_a k_b. \tag{142}$$

Choosing a Cartan basis $\{\Phi_I\}_{I=1}^3$ for $\mathfrak{u}(3)$ and then the constant Cartan connection can be written as

$$A_a = \frac{1}{2\pi}\,\Upsilon_a^{\ I}\,\Phi_I, \qquad [\Phi_I, \Phi_J] = 0, \tag{143}$$

The Wilson loop around ℓ_a is then

$$W_a = \mathcal{P}\exp\Big(-i\oint_{\ell_a} A\Big) = \exp\big(-i\,\Upsilon_a^{\ I}\Phi_I\big) \in U(3). \tag{144}$$

Let R_f be a representation of $U(3)$ and $|w\rangle$ a simultaneous eigenstate of the Cartan generators,

$$\Phi_I|w\rangle = w_I\,|w\rangle. \tag{145}$$

Then $|w\rangle$ is an eigenvector of each Wilson loop with eigenvalue

$$\begin{aligned} W_a\,|w\rangle &= e^{-i\,\Upsilon_a^{\ I} w_I}\,|w\rangle, \\ \Theta_a(w) &= \Upsilon_a^{\ I} w_I \pmod{2\pi}. \end{aligned} \tag{146}$$

For a fixed set of type covectors $k_i \in \mathbb{R}^3$, the three observed families correspond to three distinguished Cartan-weight vectors $w_{n,i} \in \mathbb{R}^3$. Any linear rule $w_{n,i} = S_n\,k_i$ with $S_n \in \mathrm{GL}(3)$ induces the factorized holonomy shifts

$$\frac{\Theta^{(n,i)}}{2\pi} = B_n k_i \pmod 1, \quad B_n = \frac{1}{2\pi}\,\Upsilon S_n \in \mathbb{R}^{3\times 3}. \tag{147}$$

For the charged fermions $n_a = 0$, where the allowed internal momenta are

$$k_a^{(n,i)} = -(B_n k_i)_a, \tag{148}$$

and the corresponding masses are therefore

$$\begin{aligned} m_{n,i}^2 = g^{ab}k_a^{(n,i)}k_b^{(n,i)} = \\ (B_n k_i)^{\mathsf{T}} g^{-1}(B_n k_i) = k_i^{\mathsf{T}} g_n k_i, \end{aligned} \tag{149}$$

where $g_n = B_n^{\mathsf{T}} g^{-1} B_n$.

In a commuting Cartan background, the internal $U(3)$ connection has gauge-invariant Wilson loops around the three non-contractible cycles. These Hosotani-Wilson [42] phases may be equivalently implemented as quasi-periodic boundary conditions for the internal wavefunctions. The holonomy generation-dependent linear map $B_n \in \mathrm{GL}(3)$ acting on type covectors $k_i \in \mathbb{R}^3$ defines the boundary twist phases which are a generation-dependent momentum $k^{(n,i)} = -B_n k_i$ on a rigid manifold $(\mathbb{T}^3, g)$, or as a family of generation-dressed, symmetric, positive-definite forms $\{g_n\}$ acting on common type covectors $\{k_i\}$.

This packaging of the Hosotani holonomy phases for charged fermions by restricting to the ground state winding provides an effective mass law and compactification on a rigid $\mathbb{T}^3$ with commuting $U(3)$ Wilson loops in which rest masses are internal Dirac eigenvalues, and the species phase vector is dressed by a generational depended mixing.

Table III shows a benchmark normal-hierarchy fit using the same internal metric that generated Table II. After fixing the observed splittings and choosing a discrete set of integer core vectors $\{\mathbf{k}_n\}$, TM yields mass eigenvalues near $m_1 = 0.026420\,\mathrm{eV}$, $m_2 = 0.027785\,\mathrm{eV}$, and $m_3 = 0.056551\,\mathrm{eV}$, with total mass $\sum_i m_i \simeq 0.111\,\mathrm{eV}$.

Table II show the fits to the known fermion masses of SM. The spectrum has the scaling redundancy $g \to s\,g$ and $k \to \frac{1}{\sqrt{s}}\,k$, which leaves $m^2 = k^{\mathsf{T}} g k$ invariant.

Fermion	SM (GeV)	TM (GeV)
e	0.000510999	0.000510987
u	0.002160000	0.002160110
d	0.004700000	0.004700050
μ	0.105658000	0.105657000
c	1.273000000	1.272990000
s	0.093500000	0.093501400
τ	1.776930000	1.776860000
t	172.5700000	172.5680000
b	4.183000000	4.183070000

TABLE II. Fit to the TM charged fermion masses using a species phase vector k_i dressed by a generation mixing matrix B_n, where the target masses are in the SM column and the TM column is the fit.

The neutrino sector is distinguished in TM by allowing a nontrivial integer winding sector that cores the dressed lepton covector down to a small residual. Introduce an integer core vector $\mathbf{k}_n \in \mathbb{Z}^3$ for each generation and define the residual neutrino momentum by

$$\mathbf{k}_{\nu,n} = B_n\,\mathbf{k}_i - \mathbf{k}_n. \tag{150}$$

Because the charged-fermion fit fixes the dimensionless temporal manifold $(g^{-1}, B_n, \mathbf{k}_i)$ only up to an overall scale, the neutrino sector introduces a physical calibration constant α_ν converting the internal residual norm or

$$m_{\nu,n} = \alpha_\nu \sqrt{\mathbf{k}_{\nu,n}^{\mathsf{T}}\, g^{-1}\, \mathbf{k}_{\nu,n}} = \alpha_\nu\, \|\mathbf{k}_{\nu,n}\|_{g^{-1}}. \tag{151}$$

The observed splittings fix α_ν once a discrete choice of integer cores $\{\mathbf{k}_n\}$ is made

$$\begin{aligned} \Delta m_{21}^2 &= \alpha_\nu^2 \Big(\|\mathbf{k}_{\nu,2}\|_{g^{-1}}^2 - \|\mathbf{k}_{\nu,1}\|_{g^{-1}}^2 \Big), \\ \Delta m_{31}^2 &= \alpha_\nu^2 \Big(\|\mathbf{k}_{\nu,3}\|_{g^{-1}}^2 - \|\mathbf{k}_{\nu,1}\|_{g^{-1}}^2 \Big). \end{aligned} \tag{152}$$

Neutrino	SM (eV^2)	TM (eV^2)
Δm_{21}^2	0.000074	0.000074
Δm_{31}^2	0.002500	0.002500
NH	**n**	TM (eV)
ν_1	(0,-1,0)	0.026420
ν_2	(1,-1,1)	0.027785
ν_3	(-2,1,0)	0.056551

TABLE III. Fit to the known Neutrino mass splitting.

Table III shows the fit using the same metric that generated Table II. Using the neutrino mass splitting in the normal hierarchy, For the discrete winding choice and calibration fixed by the measured splittings, TM yields a lightest neutrino mass of 0.026 eV. In the Cartan quantization, the lowest lying mode is $(0,-1,0)$ for the lightest neutrino with higher windings for the two heavier neutrinos. The total mass for the neutrinos is 0.11 eV.

Neutrino masses arise from the internal temporal Dirac operator in exactly the same way other fermion masses, making neutrinos Dirac fermions. In this realization there is no fundamental Majorana mass term, and lepton number is conserved at the renormalizable level. A minimal spectral Dirac neutrino sector should make neutrinoless double beta decay absent at observable rates[43–46].

The photon and Z are the usual Cartan mixtures after symmetry breaking. The Higgs doublet is the lowest scalar excitation of the internal connection along the coset directions Π with its mass set by the internal gauge action and the volume of $\mathbb{T}^3$. TM supports a sterile right-handed neutrino sector, where trivial representation spinor modes of $U(3)$ with no T^3 and no Y, whose internal Dirac eigenvalues give gravitationally interacting fermions that can supply dark matter through an effective cosmological constant from the same trivial temporal sector. The graviton is then the fluctuation of the Lorentzian metric reconstructed from a chronometric field ϕ, and couples universally to all internal energy, including temporal curvature.

XI. MIXING AND HIGGS DYNAMICS

The previous section treated a commuting Cartan background on $\mathbb{T}^3$, which is sufficient to extract a zeroth-order spectral mass law. The present section relaxes that restriction and allows generic non-abelian holonomy. Once the Wilson loops around independent cycles fail to commute, the internal Dirac operators in different sectors are no longer simultaneously diagonalizable, and the resulting misalignment appears in four dimensions as CKM and PMNS mixing. Thus the commuting background controls leading-order mass scales, while noncommuting holonomy and coset fluctuations control flavor mixing and Higgs-induced off-diagonal structure.

Let $\mathcal{C} \subset \mathbb{T}^3$ be a closed loop and define the temporal Wilson line $W^{(f)}(\mathcal{C}) = \mathcal{P}\exp\Big(-i \oint_{\mathcal{C}} A^{(f)}\Big) \in U(3)$, where $A \in \Omega^1(\mathcal{M}_\tau, \mathfrak{u}(3))$ is the fixed internal connection. Each SM fermion sector f carries a representation $R_f : U(3) \to U(\mathbb{H}_\tau^{(f)})$, and the induced sector connection $A^{(f)} = R_{f*}(A)$, and holonomy $W^{(f)}(\mathcal{C}) = R_f\Big(\mathcal{P}e^{-i\oint_{\mathcal{C}} A}\Big) \in U(\mathbb{H}_\tau^{(f)})$. For flavor, a generic $U(3)$ background on $\mathbb{T}^3$ admits noncommuting holonomies for loops $\mathcal{C}_1, \mathcal{C}_2$. The matrices $W(\mathcal{C}_1)$ and $W(\mathcal{C}_2)$ need not commute, so there is in general no global basis that diagonalizes the temporal parallel transport.

In TM, this non-abelian holonomy is processed through the sector representations R_f and through the sector's temporal Dirac operators $D_\tau^{(f)}$, whose eigenmodes define the internal wavefunctions $\chi^{(f)}(\theta)$ that enter the effective four-dimensional theory. In the low-energy theory the fermion masses arise from Yukawa matrices Y_f and the Higgs vacuum expectation value, so that

$$M_f = \frac{v}{\sqrt{2}}\, Y_f, \tag{153}$$

The physical mass eigenstates are obtained by biunitary diagonalization

$$U_L^{(f)\dagger}\, M_f\, U_R^{(f)} = \mathrm{diag}(m_{f_1}, m_{f_2}, m_{f_3}), \tag{154}$$

where the neutrinos are just appended to Equation (132) as $f = \{u, d, e, \nu\}$, and only the left-handed matrices $U_L^{(f)}$ enter charged currents. Therefore the observable mixing matrices are the relative left-handed basis mismatches between paired sectors,

$$U_{\mathrm{PMNS}} = U_L^{(e)\dagger} U_L^{(\nu)}, \quad V_{\mathrm{CKM}} = U_L^{(u)\dagger} U_L^{(d)}. \tag{155}$$

In TM, the matrices $U_L^{(f)}$ are not free inputs. They are determined by the same underlying temporal holonomy of A evaluated in the representation R_f and restricted to the relevant left-handed subbundle selected by the $SU(2)_L$ embedding together with the sector-dependent spectrum and eigenmodes of $D_\tau^{(f)}$. Because the background holonomies are noncommuting, the induced effective Yukawa matrices need not be simultaneously diagonalizable across sectors, and CKM-PMNS [47–49] mixing emerges as a geometric consequence of a single internal $U(3)$ connection.

Within TM, the Higgs doublet is interpreted as the lowest scalar excitation associated with fluctuations of the internal $U(3)$ connection along the coset directions of the electroweak embedding described at the end of Section VI. Concretely, the internal gauge field is expanded about the locked background as

$$\begin{aligned} A_a(x,\theta) &= A_a^{(\mathrm{bg})}(\theta) + \delta A_a(x,\theta), \\ \delta A_a(x,\theta) &= \delta A_a^{(\mathrm{unbroken})}(x,\theta) + \\ &\quad \xi(x)\,\xi_a(\theta) + \xi^\dagger(x)\,\xi_a^\dagger(\theta) + \dots, \end{aligned} \tag{156}$$

where $\xi_a(\theta)$ takes values in the coset generators. So that $\xi(x)$ transforms as an $SU(2)_L$ doublet with hypercharge $+1/2$. The locking ansatz is imposed on the background fields, while Higgs dynamics arise from small x-dependent fluctuations $\delta A_a(x,\theta)$ in the traceless coset directions. These fluctuations generate mixed field strength components of linear order,

$$F_{\mu a} = \partial_\mu A_a - \partial_a A_\mu + i[A_\mu, A_a], \tag{157}$$

and their four-dimensional kinetic term descends from the higher-dimensional Yang-Mills [50] action

$$S = -\frac{1}{4g_\tau^2}\int d^4x\, d^3\theta\, \sqrt{-g_4 g_\tau}\;\mathrm{Tr}\,\mathcal{F}_{AB}\mathcal{F}^{AB}. \tag{158}$$

Separating background and fluctuations as

$$\begin{aligned} A_\mu &= A_\mu^{(\mathrm{bg})}(x) + \delta A_\mu(x,\theta), \\ A_a &= A_a^{(\mathrm{bg})}(\theta) + \delta A_a(x,\theta), \end{aligned} \tag{159}$$

the mixed curvature expands to first order as

$$\mathcal{F}_{\mu a} = \partial_\mu \delta A_a - D_a^{(\mathrm{bg})}\delta A_\mu + \dots, \tag{160}$$

and inserting the KK-style mode decomposition Equation (156) into $\int d^3\theta\,\mathrm{Tr}\,\mathcal{F}_{\mu a}\mathcal{F}^{\mu a}$ yields the Higgs kinetic term

$$|D_\mu \xi|^2,\; D_\mu \xi = \left(\partial_\mu + i\sum_{k\in EW} q_k A_\mu^{(k)}(x)\,T_k^{(H)}\right)\xi. \tag{161}$$

providing the geometric origin of $(D_\mu\xi)^\dagger(D^\mu\xi)$, and in the standard way with an $SU(2)_L \times U(1)_Y$ basis, $A_\mu^{(k)}T_k^{(H)} = g\,W_\mu^a T_a^{(H)} + g' B_\mu Y_H$, with $Y_H = +1/2$.

In gauge-Higgs unification the quartic interaction is inherited from the non-abelian curvature in the purely internal sector, $\mathrm{Tr}\,F_{ab}F^{ab} \supset \mathrm{Tr}[A_a, A_b]^2$. Once the coset mode is projected onto its normalized internal profile $\xi_a(\theta)$, and its effective coupling is set by g_τ. The Higgs mass term is controlled by the same geometry as in standard gauge-Higgs unification where "the Higgs boson is identified with the fifth component of a gauge field" [51], can arise from the holonomy and radiative effects consistent with the higher-dimensional gauge symmetry. TM treats the electroweak Higgs as an internal component of the $U(3)$ temporal connection with its dynamics emerging from the time-bundle curvature rather than from an independent scalar sector.

The same identification also ties Yukawa couplings directly to geometry. Starting from the fermion action

$$S = \int d^4x\, d^3\theta\, \sqrt{-g_4 g_\tau}\;\bar{\Psi}\big(i\Gamma^\mu D_\mu + i\Gamma^a D_a\big)\Psi, \tag{162}$$

with

$$\begin{aligned} D_\mu &= \partial_\mu + \omega_\mu + iA_\mu(x,\theta), \\ D_a &= \partial_a + \omega_a + iA_a(x,\theta), \end{aligned} \tag{163}$$

and the product representation $\Gamma^\mu = \gamma^\mu \otimes \mathbf{1}$ and $\Gamma^a = \gamma^5 \otimes \gamma^a$ with the internal gauge interaction contains

$$S_{\mathrm{int}} \supset \int d^4x\,\sqrt{-g_4}\int d^3\theta\,\sqrt{g_\tau}\;\bar{\Psi}(x,\theta)\,i\Gamma^a A_a(x,\theta)\,\Psi(x,\theta). \tag{164}$$

Expanding the fermions into four-dimensional fields times temporal eigenmodes,

$$\begin{aligned} \Psi(x,\theta) \sim &\sum_i \psi_{L,i}^{(f)}(x)\chi_{L,i}^{(f)}(\theta) + \\ &\sum_j \psi_{R,j}^{(f)}(x)\chi_{R,j}^{(f)}(\theta) + \dots, \end{aligned} \tag{165}$$

and inserting the coset fluctuation $\delta A_a(x,\theta) \supset \xi(x)\,\xi_a(\theta) + \xi^\dagger(x)\,\xi_a^\dagger(\theta)$ gives

$$\begin{aligned} S_{\mathrm{int}} = &\int d^4x\,\sqrt{-g_4}\sum_{i,j}\bar{\psi}_{L,i}^{(f)}(x)\,\xi(x)\,\psi_{R,j}^{(f)}(x) \\ &\left[\int d^3\theta\,\sqrt{g_\tau}\;\chi_{L,i}^{(f)\dagger}(\theta)\,(i\Gamma^a)\,\xi_a(\theta)\,\chi_{R,j}^{(f)}(\theta)\right] + \cdots. \end{aligned} \tag{166}$$

which reduces to the four-dimensional Yukawa interaction

$$S_{\mathrm{int}} = \int d^4x\,\sqrt{-g_4}\;\sum_{i,j}\bar{\psi}_{L,i}^{(f)}(x)\,\xi(x)\,\psi_{R,j}^{(f)}(x)\;(Y_f)_{ij} + \cdots, \tag{167}$$

with the geometric Yukawa matrix given by the overlap integral

$$(Y_f)_{ij} = i\int_{\mathcal{M}_\tau} d^3\theta\,\sqrt{g_\tau}\;\chi_{L,i}^{(f)\,\dagger}(\theta)\,\Gamma^a\xi_a(\theta)\,\chi_{R,j}^{(f)}(\theta). \tag{168}$$

This recovers the Yukawa Lagrangian,

$$\begin{aligned} \mathcal{L}_Y^{(f)} = &-(Y_f)_{ij}\,\bar{\psi}_{L,i}^{(f)}(x)\,\xi(x)\,\psi_{R,j}^{(f)}(x) \\ &-(Y_f)_{ij}^*\,\bar{\psi}_{R,j}^{(f)}(x)\,\xi^\dagger(x)\,\psi_{L,i}^{(f)}(x), \end{aligned} \tag{169}$$

and makes explicit why flavor structure is not independent in TM. Hierarchies in fermion masses correspond to hierarchies in internal overlaps of temporal eigenmodes with the Higgs profile. Complex phases arise from nontrivial holonomy and internal mode structure. The resulting Yukawa matrices are not constrained to be symmetric and the same coset fluctuation that generates $(D_\mu\xi)^\dagger(D^\mu\xi)$ also generates the Yukawa sector, while the noncommuting temporal holonomy that controls the $\chi^{(f)}(\theta)$ ensures that the Y_f are misaligned across sectors, producing CKM-PMNS mixing.

XII. CONCLUSION

TM is a shift in what is taken as fundamental. Instead of quantizing spacetime, TM elevates quantum structure to temporal geometry with quantized windings. The $U(1)$ phase that appears as Schrödinger time evolution is interpreted as winding and its dual in internal time, and the unitary phase evolution in external time is a shadow of that winding. In this sense quantum mechanics is temporal in nature, while relativity governs how those temporal structures are connected through proper time and the Lorentz group.

External Lorentzian geometry is sourced by effective stress-energy induced by internal temporal structure. In this sense, gravitational dynamics are not independent of temporal geometry but are its external geometric response. Small perturbations of the emergent metric are therefore interpreted as the usual diffeomorphism-governed spin-2 excitations of the effective spacetime description, rather than as independent fundamental degrees of freedom prior to the temporal construction.

Once the $U(3)$ temporal bundle is fixed in the construction with $\mathcal{M}_\tau = \mathbb{T}^3$, SM structures become different projections of the same temporal geometry. Rest masses arise spectrally as eigenvalues of the internal Dirac operator D_τ. Flavor mixing is holonomy, where CKM-PMNS originate from the same non-abelian parallel transport in the temporal bundle evaluated in different representations and chiral restrictions. The observed electroweak scalar is interpreted geometrically as the lowest coset excitation of the internal connection along Π, rather than a fundamental scalar postulated to solve the hierarchy problem. Electroweak structure and the Higgs coset arise from the $U(3)$ sector on E. $SU(3)_C$ is carried by the Hodge factor $SU(3)_C \subset U(3)$ acting on the negative generalized subbundle C.

The apparent arrow of time and quantum measurement are interpreted as projections of coherent $U(1)$ evolution in internal time, where perceived time emerges from the orientation and monotonic phase winding of internal $U(1)$ evolution over $\mathcal{M}_\tau$, inducing geodesic structure in external spacetime. TM recasts physics as manifestations of spectral and topological structure on a temporal manifold, unifying gauge theory, general relativity, and quantum field theory within a single geometric paradigm.

In terms of cosmological implications, it is possible that the same internal temporal geometry that fixes the four-dimensional mass spectrum also partitions cosmological components into sectors of the temporal spectrum. After the phase lock, the internal spinor modes that lie in the trivial-representation sector of the temporal $U(3)$ bundle and carry no $SU(3)_C \times SU(2)_L \times U(1)_Y$ charges appear in four dimensions as gauge-inert fermions, providing a dark sector whose masses are set by the trivial-sector eigenvalues of the internal temporal Dirac operator and whose cosmological impact is governed by their phase-space distributions and production history.

This would make dark components which are not separate substances appended to gravity but distinct sectors of the temporal spectrum. Visible matter corresponds to nontrivial representations, while trivial modes supply a generic sterile population that could constitute dark matter. Depending on their characteristic momentum at production, these species could behave as cold, warm, or hot relics where cold components behave as standard collisionless dark matter. In the rigid late-time regime assumed, temporal vacuum energy associated with the internal geometry mimic a cosmological constant. The same geometric overlap structure that yields effective Yukawa couplings in the visible sector also enables early-universe population of the trivial sector.

[1] A. Einstein, The foundation of the general theory of relativity, Annalen der Physik **49**, 770 (1916).

[2] A. Einstein, Zur Elektrodynamik bewegter Körper, Annalen der Physik **322**, 891 (1905).

[3] R. M. Wald, *General Relativity* (University of Chicago Press, 1984).

[4] S. M. Carroll, *Spacetime and Geometry: An Introduction to General Relativity* (Addison Wesley, San Francisco, 2004).

[5] G. Whitrow, *Time in History: Views of Time from Prehistory to the Present Day* (Oxford University Press, 1989).

[6] A. D. Ludlow, M. M. Boyd, J. Ye, E. Peik, and P. O. Schmidt, Optical atomic clocks, Rev. Mod. Phys. **87**, 637 (2015).

[7] E. Noether, Invariante variationsprobleme, Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen, Mathematisch-Physikalische Klasse **1918**, 235 (1918).

[8] J. J. Sakurai and J. Napolitano, *Modern Quantum Mechanics*, 2nd ed. (Cambridge University Press, 2017).

[9] M. Nakahara, *Geometry, Topology and Physics*, 2nd ed. (CRC Press, 2003).

[10] J. C. Baez and J. P. Muniain, *Gauge Fields, Knots and Gravity* (World Scientific, 1994).

[11] T. Frankel, *The Geometry of Physics: An Introduction* (Cambridge University Press, 2011).

[12] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (CRC Press, 2018).

[13] S. Weinberg, *The Quantum Theory of Fields* (Cambridge University Press, 1995).

[14] P. A. M. Dirac, Quantised singularities in the electromagnetic field, Proc. R. Soc. Lond. A **133**, 60 (1931).

[15] T. T. Wu and C. N. Yang, Concept of nonintegrable phase factors and global formulation of gauge fields, Phys. Rev. D **12**, 3845 (1975).

[16] S. S. Chern, *Complex Manifolds Without Potential Theory* (Springer, 1979).

[17] N. M. J. Woodhouse, *Geometric Quantization* (Oxford University Press, 1992).

[18] V. Fock, Zur schrödingerschen wellenmechanik,

Zeitschrift für Physik **38**, 242 (1926).
[19] H. Weyl, Elektron und gravitation. i, Zeitschrift für Physik **56**, 330 (1929).
[20] F. London, Quantenmechanische deutung der theorie von weyl, Zeitschrift für Physik **42**, 375 (1927).
[21] T. Kaluza, Zum unitätsproblem der physik, Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.) , 966–972 (1921).
[22] O. Klein, Quantum theory and five-dimensional theory of relativity, Zeitschrift für Physik **37**, 895–906 (1926).
[23] M. J. Duff, Kaluza–klein theory in perspective, in *The Oskar Klein Centenary Nobel Symposium* (Stockholm, Sweden, 1994) pp. 22–35, arXiv:hep-th/9410046.
[24] H. Georgi, *Lie Algebras in Particle Physics*, 2nd ed. (Westview Press, Boulder, CO, 1999).
[25] M. Gell-Mann, Symmetries of baryons and mesons, Physical Review **125**, 1067 (1962), introduces the eightfold way and the $\mathfrak{su}(3)$ generator structure commonly represented with the Gell-Mann matrices.
[26] J. Milnor and J. D. Stasheff, *Characteristic Classes* (Princeton University Press, Princeton, 1974).
[27] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry, Vol. 1* (Interscience Publishers, New York, 1963).
[28] N. Steenrod, *The Topology of Fibre Bundles* (Princeton University Press, Princeton, 1951).
[29] C. Hull and B. Zwiebach, The gauge algebra of double field theory and courant brackets, J. High Energy Phys. **2009** (09), 090.
[30] O. Hohm and B. Zwiebach, On the riemann tensor in double field theory, J. High Energy Phys. **2012** (05), 126.
[31] C. Hull and B. Zwiebach, Double field theory, J. High Energy Phys. **2009** (09), 099.
[32] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, 1973).
[33] H. Georgi and S. L. Glashow, Unity of all elementary-particle forces, Phys. Rev. Lett. **32**, 438 (1974).
[34] J. C. Baez and J. Huerta, The algebra of grand unified theories, Bulletin of the American Mathematical Society **47**, 483 (2010).
[35] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Theory and phenomenology of two-Higgs-doublet models, Phys. Rept. **516**, 1 (2012).
[36] G. Aad *et al.* (ATLAS), Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys. Lett. B **716**, 1 (2012).
[37] S. Chatrchyan *et al.* (CMS), Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys. Lett. B **716**, 30 (2012).
[38] A. Collaboration, *A combination of measurements of Higgs boson production and decay using up to 139 fb^{-1} of proton–proton collision data at $\sqrt{s} = 13$ TeV collected with the ATLAS experiment*, Tech. Rep. ATLAS-CONF-2021-053 (CERN, 2021).
[39] H. Weyl, Gravitation und elektrizität, Sitzungsberichte der Preussischen Akademie der Wissenschaften, Physikalisch-Mathematische Klasse , 465 (1918), foundational paper introducing local scale (Weyl) invariance; origin of "Weyl transformations".
[40] B. Pontecorvo, Neutrino experiments and the problem of conservation of leptonic charge, Soviet Physics JETP **26**, 984 (1968).
[41] F. Bloch, Über die quantenmechanik der elektronen in kristallgittern, Zeitschrift für Physik **52**, 555 (1929).
[42] Y. Hosotani, S. Noda, and K. Takenaga, Dynamical gauge symmetry breaking and mass generation on the orbifold t^2/z_2, Phys. Rev. D **69**, 125014 (2004).
[43] A. Gando *et al.* (KamLAND-Zen), Search for Majorana Neutrinos near the Inverted Mass Hierarchy Region with KamLAND-Zen, Phys. Rev. Lett. **117**, 082503 (2016).
[44] M. Agostini *et al.* (GERDA), Final Results of GERDA on the Search for Neutrinoless Double-β Decay, Phys. Rev. Lett. **125**, 252502 (2020).
[45] D. Q. Adams *et al.* (CUORE), Search for Majorana neutrinos exploiting the complete CUORE dataset, Phys. Rev. Lett. **128**, 022501 (2022).
[46] J. B. Albert *et al.* (EXO-200), Search for Neutrinoless Double-Beta Decay with the Complete EXO-200 Dataset, Phys. Rev. Lett. **123**, 161802 (2019).
[47] N. Cabibbo, Unitary symmetry and leptonic decays, Phys. Rev. Lett. **10**, 531 (1963).
[48] M. Kobayashi and T. Maskawa, Cp-violation in the renormalizable theory of weak interaction, Prog. Theor. Phys. **49**, 652 (1973).
[49] Z. Maki, M. Nakagawa, and S. Sakata, Remarks on the unified model of elementary particles, Prog. Theor. Phys. **28**, 870 (1962).
[50] C. N. Yang and R. L. Mills, Conservation of isotopic spin and isotopic gauge invariance, Phys. Rev. **96**, 191 (1954).
[51] J. Yoon and M. E. Peskin, Dissection of an SO(5)×U(1) gauge–higgs unification model, Phys. Rev. D **100**, 015001 (2019).